2D Perimeter & Area

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▷ Perimeter vs. Area

Perimeter = total length of the **boundary** (units: cm, m, ...)

Area = amount of **surface** enclosed (units: cm^2, m^2, ...)

▷ Perimeter = the *fence* around a garden

▷ Area = the *grass* inside the garden

Never confuse them — check units in your answer!

◇ Unit Conversions

Length: $1\,\text{cm} = 10\,\text{mm}$ $1\,\text{m} = 100\,\text{cm}$ $1\,\text{km} = 1000\,\text{m}$

Area (square the conversion factor!):

- $1\,\text{cm}^2 = \mathbf{100}\,\text{mm}^2$ (because $10^2 = 100$)
- $1\,\text{m}^2 = \mathbf{10{,}000}\,\text{cm}^2$ (because $100^2 = 10{,}000$)
- $1\,\text{km}^2 = \mathbf{1{,}000{,}000}\,\text{m}^2$ (because $1000^2 = 10^6$)

Never write $1\,\text{m}^2 = 100\,\text{cm}^2$!

★ Formula Reference Table

Shape	Key dimensions	Perimeter	Area
Rectangle	length ℓ, width w	$2(\ell + w)$	ℓw
Square	side s	$4s$	s^2
Triangle	base b, perp. height h, sides a, b, c	$a + b + c$	$\frac{1}{2}bh$
Parallelogram	base b, perp. height h, slant side a	$2(a + b)$	bh
Trapezium	parallel sides a, b; height h; legs c, d	$a + b + c + d$	$\frac{1}{2}(a + b)h$
Circle	radius r (diameter $d = 2r$)	$2\pi r = \pi d$	πr^2
Semicircle	radius r	$\pi r + 2r$	$\frac{1}{2}\pi r^2$
Sector (θ°)	radius r, central angle θ	$\frac{\theta}{360} \times 2\pi r + 2r$	$\frac{\theta}{360} \times \pi r^2$

◯ Circles & Sectors

Finding a missing dimension:

- Given $C \to r = \dfrac{C}{2\pi}$
- Given $A \to r = \sqrt{\dfrac{A}{\pi}}$
- Given $d \to$ **halve first**: $r = \dfrac{d}{2}$

Sectors = fraction $\dfrac{\theta}{360}$ of a full circle:

$$L = \tfrac{\theta}{360} \times 2\pi r \qquad A_{\text{sec}} = \tfrac{\theta}{360} \times \pi r^2$$

Shortcut (given A and r, find arc L):

$$L = \frac{2A}{r}$$

Sector perimeter $= L + 2r$ (arc **plus** both radii)

⊞ Composite & Shaded Shapes

Composite shapes (addition):

1. Split into basic shapes
2. Find each area separately
3. **Add** the areas together
4. **Perimeter** = outer edges only (internal split lines do **not** count!)

Shaded regions (subtraction):

$$\text{Shaded} = A_{\text{outer}} - A_{\text{inner}}$$

Uniform border of width x around $\ell \times w$:

$$A_{\text{border}} = (\ell + 2x)(w + 2x) - \ell w$$

△ Common Traps & Pitfalls

1. **Height ≠ slant side!** Always use the **perpendicular** height in $A = bh$ and $A = \frac{1}{2}bh$
2. **Radius ≠ diameter!** If given d, write $r = d/2$ before substituting
3. **Area units need squaring:** $1\,\text{m}^2 = 10{,}000\,\text{cm}^2$, **not** $100\,\text{cm}^2$
4. **Semicircle perimeter includes diameter:** $P = \pi r + 2r$ (curved arc **plus** straight edge)
5. **Internal edges don't count** in perimeter of composite shapes

✓ Quick Exam Strategy

- **Step 1:** Read carefully — perimeter or area?
- **Step 2:** Check units — convert **before** substituting
- **Step 3:** Identify the shape(s) and select formula
- **Step 4:** For composite → split; for shaded → subtract
- **Step 5:** Keep π exact until the very last step
- **Step 6:** Write units! (cm vs cm^2)
- **"Find r given area/circumference"** → rearrange the formula, then substitute
- **State reasons. Sketch the shape. Check!**

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