3D Surface Area & Volume

Prepared by Nikolas Agisilaou

Key Definitions

Surface area = total area of all outer faces (in **square** units: cm^2, m^2).

Volume = the space a solid occupies (in **cubic** units: cm^3, m^3).

Net = flat 2D shape that folds into the solid; its total area = surface area.

Think: SA = wrapping paper; V = what fits inside.

Unit Conversions

Volume conversions are cubed!

- $1\,\text{cm}^3 = 1000\,\text{mm}^3$
- $1\,\text{m}^3 = 1{,}000{,}000\,\text{cm}^3$

Capacity:

- $1\,\text{cm}^3 = 1\,\text{ml}$; $1000\,\text{cm}^3 = 1\,\text{litre}$
- $1\,\text{m}^3 = 1000\,\text{litres}$

Master Formula Table

Shape	Volume	Surface Area
Cube (s)	s^3	$6s^2$
Cuboid	ℓwh	$2(\ell w+\ell h+wh)$
Prism	$A \times L$	$2A + PL$
Cylinder	$\pi r^2 h$	$2\pi r(h + r)$

Shape	Volume	Surface Area
Pyramid	$\frac{1}{3}Ah$	Base + tri. faces
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r(l + r)$
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Hemisphere	$\frac{2}{3}\pi r^3$	$3\pi r^2$

A = cross-section area, P = perimeter, L = length, l = slant height. CSA: cylinder $= 2\pi rh$, cone $= \pi rl$, hemisphere $= 2\pi r^2$.

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Key Relationships & Rearrangements

The One-Third Rule:

- $V_{\text{cone}} = \frac{1}{3} V_{\text{cylinder}}$ *(same base & height)*
- $V_{\text{pyramid}} = \frac{1}{3} V_{\text{prism}}$ *(same base & height)*

Pythagoras link:

- Cone: $l^2 = r^2 + h^2$
- Square pyramid: $l^2 = h^2 + (a/2)^2$

Common rearrangements:

Find r from V (cyl) $r = \sqrt{V/(\pi h)}$

Find h from V (cyl) $h = V/(\pi r^2)$

Find r from SA (sph) $r = \sqrt{SA/(4\pi)}$

Find r from V (sph) $r = \sqrt[3]{3V/(4\pi)}$

Composite Solids: Strategy

A **composite solid** = two or more basic shapes joined (or subtracted). Follow this approach:

1. **Identify** the basic shapes that make up the solid (e.g. cylinder + hemisphere, cylinder + cone).
2. **Volume:** Add (or subtract) volumes of each part. Volume is **conserved** in melting/recasting.
3. **Surface area:** Add exposed surfaces, but **subtract hidden (internal) faces** where shapes meet.
4. **Melting/recasting:** Set $V_{\text{original}} = n \times V_{\text{new}}$ and solve for n (round **down** for complete objects).

Quick example: Cyl (r, h) + hemisphere (r) on top $\Rightarrow$ $V = \pi r^2 h + \frac{2}{3}\pi r^3$ | $SA = 3\pi r^2 + 2\pi rh$ *(top circle hidden)*

Common Traps & Pitfalls

1. **Radius vs. Diameter:** Given the diameter? **Halve it** before using any formula. This catches many students out.
2. **Perpendicular height vs. Slant height:** Use h (perpendicular) for **volume**; use l (slant) for **surface area**. Convert between them with Pythagoras.
3. **Volume unit conversions are cubed:** $1\,\text{m}^3 = 1{,}000{,}000\,\text{cm}^3$ (not 100!). Cube the linear factor.
4. **Hemisphere total SA includes the flat base:** $SA = 2\pi r^2 + \pi r^2 = 3\pi r^2$, not just $2\pi r^2$.
5. **Hidden faces in composites:** Where two solids join, the contact faces are internal — subtract them from SA.
6. **Premature rounding:** Keep π exact throughout your working. Only round in the **final** answer.

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Exam Checklist & Quick Tips

Before you calculate: Read carefully → Volume or SA? → Sketch & label → Same units? → Pick the formula.

What exams test most:

1. Standard V & SA of single solids
2. Prism: $V = A \times L$
3. Composite solids (2+ shapes)
4. Melting / recasting (volume conserved)
5. Unit conversions ($\text{cm}^3 \leftrightarrow$ litres)
6. Finding missing dimensions from V or SA

Decision checklist: Prism? → Find cross-section. Pyramid/Cone? → Use $\frac{1}{3}$. Hemisphere? → Add base. Composite? → Hidden faces!