Algebraic Fractions

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Algebraic Fractions (What They Are)

A fraction whose numerator, denominator, or both are algebraic expressions.

$$\frac{3p}{7q}, \quad \frac{x^2-25}{x+5}, \quad \frac{2x^2+9x+4}{x^2+2x-8}$$

- **To simplify:** write in lowest terms by cancelling common **factors**.
- **Factor:** connected by $\times$ (can cancel).
- **Term:** connected by $+$ or $-$ (**cannot** cancel).

Simplifying: The Method

Always factorise FIRST, cancel SECOND.

1. Factorise the **numerator** completely.
2. Factorise the **denominator** completely.
3. Cancel any factor appearing in **both**.
4. Write result. No common factor $\rightarrow$ already simplified.

Key: Factorise $\rightarrow$ Cancel $\rightarrow$ Write

The Four Operations at a Glance

Operation	Formula / Rule	Key Steps
Simplify	Cancel common **factors** only	Factorise numerator & denominator $\rightarrow$ cancel matching factors $\rightarrow$ write result
Multiply	$\frac{A}{B} \times \frac{C}{D} = \frac{AC}{BD}$	Factorise all expressions $\rightarrow$ cancel any factor on top *and* bottom $\rightarrow$ multiply remaining
Divide	$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$	**K**eep first $\rightarrow$ **C**hange $\div$ to $\times$ $\rightarrow$ **F**lip second $\rightarrow$ multiply as above
Add/Subtract	$\frac{A}{B} \pm \frac{C}{D} = \frac{AD \pm BC}{BD}$	Find LCD $\rightarrow$ rewrite fractions over LCD $\rightarrow$ combine numerators $\rightarrow$ bracket when subtracting $\rightarrow$ simplify

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Adding & Subtracting: Choosing the LCD

Denominator situation	LCD to use
Same denominator	Combine numerators directly
Numerical only (e.g. $\frac{x}{4}+\frac{y}{5}$)	LCM of the numbers (e.g. 20)
One factorises into the other	Use the larger / more complex one
No common factors	Multiply the denominators together

Add/Subtract Steps

1. Factorise both denominators
↓
2. Identify the LCD
↓
3. Rewrite each fraction over LCD
↓
4. Expand & combine
(use **brackets** when subtracting!)
↓
5. Factorise numerator & cancel

Solving Equations with Algebraic Fractions

Goal: eliminate all fractions by multiplying through by the LCD, then solve normally.

1. Find the LCD of *all* denominators in the equation.

2. Multiply **every term** on both sides by the LCD.

3. Simplify — all fractions should disappear completely.

4. Solve the resulting linear or quadratic equation.

5. **CHECK:** substitute each answer back into the **original** equation.
Reject any value that makes a denominator zero.

Extraneous solutions: multiplying by an expression in x can generate extra roots that are undefined in the original. **Always check every solution.**

Common Traps & Pitfalls

1. **The Terms Trap:** $\dfrac{x+3}{x+5} \neq \dfrac{3}{5}$. You **cannot** cancel x across a $+$ sign — x is a *term*, not a factor. Always factorise first; only cancel complete matching factors.
2. **The Bracket Trap:** Subtracting $\frac{2x+9}{x-3}$ gives $\frac{-(2x+9)}{x-3} = \frac{-2x-9}{x-3}$, **not** $\frac{-2x+9}{x-3}$. A minus reverses **every** term inside the bracket.
3. **The LCD Trap:** Never assume LCD = product of denominators. Factorise first to check for shared factors. e.g. $(x+2)$ and $(x+2)(x-3)$ have LCD $= (x+2)(x-3)$, not their product.
4. **The KCF Trap:** $\dfrac{A}{B} \div \dfrac{C}{D}$: only **flip the second** fraction to get $\dfrac{A}{B} \times \dfrac{D}{C}$. Never flip the first fraction or both fractions.
5. **The Extraneous Trap:** A value may satisfy the rearranged equation yet zero a denominator

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in the original (e.g. $x = 2$ making $x - 2 = 0$). Always test in the **original** equation before writing your answer.

Quick Exam Shortcuts

Rate problems: "A alone takes m hrs, B alone takes n hrs, together T hrs" $\Rightarrow$ $\dfrac{1}{m} + \dfrac{1}{n} = \dfrac{1}{T}$.

Distance–Speed–Time: "same distance d, speeds v_1 and v_2, time difference Δt" $\Rightarrow$ $\dfrac{d}{v_1} - \dfrac{d}{v_2} = \Delta t$ (or $+\Delta t$ if slower takes longer).

"Show that..." questions: work backwards from the given target; find the LCD and verify the numerator matches after full expansion.

Perfect simplification: if everything cancels to a constant (e.g. just 3), that is completely correct — do **not** write it as $\frac{3}{1}$.

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