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Completing the Square

IGCSE Mathematics

The Panda Math Tutor

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1 Introduction

Note

Completing the square is a powerful algebraic technique used to rewrite any quadratic expression in a more useful form. It is one of the most important skills in IGCSE Mathematics and forms the foundation for understanding quadratic graphs, solving equations, and deriving the quadratic formula itself.
In this chapter, we will explore:

- Perfect square identities and why they matter
- Completing the square when the coefficient of x^2 is 1
- Completing the square when the coefficient of x^2 is not 1
- Solving quadratic equations by completing the square
- Deriving the quadratic formula

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2 Perfect Square Identities

Before learning the technique, it is essential to recognise and expand perfect square expressions with confidence.

Perfect Square Identities

The two fundamental perfect square identities are:

$$(x+a)^2 = x^2 + 2ax + a^2$$
$$(x-a)^2 = x^2 - 2ax + a^2$$

These identities are the key to understanding how completing the square works.

Example

Expand each of the following:

(a) $(x+7)^2$ (b) $(x-6)^2$ (c) $\left(x+\frac{3}{2}\right)^2$

Solution:

(a) $(x+7)^2 = x^2 + 2(7)x + 7^2 = x^2 + 14x + 49$
(b) $(x-6)^2 = x^2 - 2(6)x + 6^2 = x^2 - 12x + 36$
(c) $\left(x + \frac{3}{2}\right)^2 = x^2 + 2 \cdot \frac{3}{2} \cdot x + \left(\frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}$

Note

Key Observation: Notice the relationship between the coefficient of x and the constant term in each expansion:

$$x^2 + 14x + \underbrace{49}_{=\left(\frac{14}{2}\right)^2} \qquad x^2 - 12x + \underbrace{36}_{=\left(\frac{-12}{2}\right)^2} \qquad x^2 + 3x + \underbrace{\frac{9}{4}}_{=\left(\frac{3}{2}\right)^2}$$

The constant term in a perfect square trinomial is always the **square of half the coefficient of** x. This observation is the entire basis of completing the square.

3 Completing the Square: Coefficient of x^2 Equal to 1

Definition

Completing the square is the process of rewriting a quadratic expression $x^2 + bx + c$ in the form:

$$(x+p)^2 + q$$

where p and q are constants.
This form is called the **completed square form** or **vertex form** of the quadratic.

Method

Completing the Square for $x^2 + bx + c$

Step 1: Write out the expression: $x^2 + bx + c$

Step 2: Take half the coefficient of x: compute $p = \frac{b}{2}$

Step 3: Write $(x+p)^2$ — this expands to $x^2 + bx + p^2$

Step 4: Subtract p^2 to correct the constant: $(x+p)^2 - p^2 + c$

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