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Completing the Square

Solutions & Answer Key

Full worked solutions, marking scheme & answer key

IGCSE Mathematics

The Panda Math Tutor

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Companion to the *Student Worksheet*

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Note

About this booklet. These are the complete solutions to every question in the *Completing the Square — Student Worksheet*. Question numbers match exactly (PS1–PS7, U1–U8, N1–N8, V1–V7, S1–S8, D1–D7, QF1–QF5, the Mixed Review MR1–MR10, and the examination questions Q1–Q8).
Every completed square answer is verified by expanding it back out, because that is exactly the habit the student should be building. Part V collects the common mistakes to watch for, and Part VI is a quick answer key for fast marking.

4 Part IV — Complete Solutions

Note

Throughout, the working follows the same three moves: halve the coefficient of x, square it, and subtract it again. When the coefficient of x^2 is not 1, that factor comes out of the first *two* terms first and must be multiplied back through at the end.

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4.1 Solutions — Perfect Square Identities

Solution PS1

Using $(x+a)^2 = x^2 + 2ax + a^2$ with $a = 9$:

$$(x+9)^2 = x^2 + 2(9)x + 9^2 = x^2 + 18x + 81.$$

Note that the constant 81 is the square of half of 18 — the relationship that makes the whole method work.

Solution PS2

With $a = 4$ and a minus sign:

$$(x-4)^2 = x^2 - 8x + 16.$$

Solution PS3

With $a = \frac{5}{2}$:

$$\left(x + \frac{5}{2}\right)^2 = x^2 + 2 \cdot \frac{5}{2} \cdot x + \left(\frac{5}{2}\right)^2 = x^2 + 5x + \frac{25}{4}.$$

The middle coefficient is 5 because the 2 cancels the denominator.

Solution PS4

In each case the constant is the square of half the coefficient of x.

(a) Half of 16 is 8, and $8^2 = 64$. $x^2 + 16x + \mathbf{64} = (x+8)^2$.

(b) Half of -22 is -11, and $(-11)^2 = 121$. $x^2 - 22x + \mathbf{121} = (x-11)^2$.

(c) Half of 7 is $\frac{7}{2}$, and $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$. $x^2 + 7x + \frac{\mathbf{49}}{\mathbf{4}} = \left(x + \frac{7}{2}\right)^2$.

Solution PS5

Expanding the right-hand side: $(x+p)^2 = x^2 + 2px + p^2$.
Compare the x terms: $2p = -14$, so $\mathbf{p = -7}$.
Compare the constants: $q = p^2 = (-7)^2 = \mathbf{49}$.
Check: $x^2 - 14x + 49 = (x-7)^2$, and $(x + (-7))^2 = (x-7)^2$. ✓

Solution PS6

Half of 12 is 6, and $6^2 = 36$. For a perfect square the constant would have to be 36, but the expression has 40 instead. Since $40 \neq 36$, the trinomial is **not** a perfect square.
The constant needed is 36, giving $x^2 + 12x + 36 = (x+6)^2$.
(In completed square form the original is $x^2 + 12x + 40 = (x+6)^2 + 4$ — the extra 4 is precisely the amount by which it fails to be a perfect square.)

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