Curved Graphs

Prepared by Nikolas Agisilaou

Quadratic Graphs (Parabolas)

Form: $y = ax^2 + bx + c, \ a \neq 0$

- **Vertex:** $x = -b/2a$; **y-intercept:** $(0, c)$
- **Vertex form:** $y = a(x-h)^2 + k \Rightarrow$ vertex (h, k)
- $a>0$: **U-shape (min)**; $a<0$: **n-shape (max)**
- **Roots:** solve $y = 0$ (0, 1, or 2 real solutions)
- **Complete square:** $y = (x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c$

Reciprocal Graphs (Hyperbolas)

Form: $y = k/x, \ k \neq 0$

- **Asymptotes:** $x = 0$ (vertical) & $y = 0$ (horizontal)
- $k>0$: branches in **Q1 & Q3**
- $k<0$: branches in **Q2 & Q4**
- $x = 0$ is **undefined**: leave a gap in all tables
- **NEVER** connect the two branches through the origin

Derivatives & The Power Rule

Rule: $\frac{d}{dx}(ax^n) = anx^{n-1}$ *"Multiply by the power; reduce it by 1"*

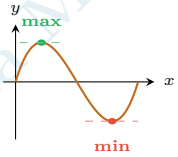
- Constants vanish: $d(c)/dx = 0$; differentiate each term separately
- **E.g.:** $y = 4x^3 - 7x^2 + 5 \Rightarrow dy/dx = 12x^2 - 14x$

y	dy/dx
c	0
x^2	$2x$
x^3	$3x^2$
ax^n	anx^{n-1}

Turning Points & Lines at a Curve

Turning (Stationary) Points

1. Set $dy/dx = 0$; solve for x
2. Substitute back to find y-coordinate
3. $d^2y/dx^2 > 0 \Rightarrow$**min**; $d^2y/dx^2 < 0 \Rightarrow$**max**



Tangent & Normal

- Tangent: $m_T = dy/dx|_{\text{point}}$
- Normal: $m_N = -1/m_T$
- **Check:** $m_T \times m_N = -1$
- Line: $y - y_1 = m(x - x_1)$

Common Traps & Pitfalls

1. **Smooth curves only:** Never join plotted points with straight line segments – always draw a free-flowing curve.
2. **Two separate branches:** Reciprocal graphs — NEVER connect them or draw a single S-curve through the origin.
3. **Tangent ≠ Normal:** $m_T = dy/dx$; $m_N = -1/m_T$; they are perpendicular so $m_T \times m_N = -1$.
4. **Turning point trap:** Set $dy/dx = 0$ (*gradient* equals zero), not $y = 0$ (which gives roots, not turning points).

Graph Shapes & Quick Exam Shortcuts

Type	Form	Key Feature
Linear	$y = mx + c$	Straight line
Quadratic	$y = ax^2 + \ldots$	Parabola; U ($a>0$) or n ($a<0$)
Cubic	$y = ax^3 + \ldots$	S-shape; no axis of symmetry
Reciprocal	$y = k/x$	2-branch hyperbola; asym. x=0, y=0
Exponential	$y = a^x$, $a>0$	Through $(0, 1)$; always above x-axis

Graphical solutions:

- $f(x) = 0$: read x-intercepts of the curve
- $f(x) = k$: draw $y = k$; read intersections
- *Related-eq. trick:* rearrange until one side equals $f(x)$; draw the other side as a straight line

Gradient shortcut: Use the *largest* possible triangle on the tangent; read only exact grid coordinates. Verify: drawn gradient ≈ dy/dx at same x.

The Panda Math Tutor