Fraction Equivalence & Operations

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What is a Fraction?

$\frac{a}{b}$: a = **numerator**, b = **denominator** ($b \neq 0$)

- **Proper:** $a < b$ (less than 1), e.g. $\frac{3}{7}$
- **Improper:** $a \geq b$ (at least 1), e.g. $\frac{11}{4}$
- **Mixed number:** whole + fraction, e.g. $2\frac{3}{4}$

Converting Mixed $\longleftrightarrow$ Improper

Mixed → Improper: $a\frac{b}{c} = \frac{a \times c + b}{c}$

e.g. $3\frac{2}{5} = \frac{3\times5+2}{5} = \frac{17}{5}$

Improper → Mixed: Divide top by bottom. Quotient = whole, remainder = numerator.

e.g. $\frac{23}{5}$: $23 \div 5 = 4 \text{ r } 3 \Rightarrow 4\frac{3}{5}$

Equivalent Fractions

Multiply or divide **both** top and bottom by the same $n \neq 0$:

$$\frac{a}{b} = \frac{a \times n}{b \times n} = \frac{a \div n}{b \div n}$$

e.g. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$

Cross-multiplication check:

$\frac{a}{b} = \frac{c}{d}$ iff $a \times d = b \times c$

Simplifying Fractions

Simplest form: HCF of top and bottom is 1.

1. Find the **HCF** of numerator and denominator
2. Divide both by the HCF

e.g. $\frac{42}{98}$: HCF $= 14 \Rightarrow \frac{42\div14}{98\div14} = \frac{3}{7}$

Never cancel individual digits — divide the *whole* top and bottom.

Comparing & Ordering Fractions

1. Find the **LCM** of the denominators
2. Convert each to equivalent fractions with that common denominator
3. Compare numerators — bigger = larger fraction

e.g. $\frac{5}{6}$ vs $\frac{7}{9}$: LCM $= 18$

$\frac{5}{6} = \frac{15}{18}$, $\frac{7}{9} = \frac{14}{18} \Rightarrow \frac{5}{6} > \frac{7}{9}$

Adding & Subtracting Fractions

Same denom: $\frac{a}{c} \pm \frac{b}{c} = \frac{a\pm b}{c}$

Different denoms:

1. Find LCM of denominators
2. Convert to equivalent fractions
3. Add/subtract **numerators only**
4. Simplify the result

e.g. $\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$

Adding & Subtracting Mixed Numbers

Method: Convert to improper fractions first.

1. Convert each mixed number to improper
2. Find LCM, convert, add/subtract
3. Convert back to mixed if needed

Examples:

- $3\frac{2}{5} + 2\frac{3}{4} = \frac{17}{5} + \frac{11}{4} = \frac{68}{20} + \frac{55}{20} = 6\frac{3}{20}$
- $5\frac{1}{3} - 2\frac{5}{6} = \frac{16}{3} - \frac{17}{6} = \frac{32}{6} - \frac{17}{6} = 2\frac{1}{2}$

Common Mistakes to Avoid

- **Adding tops & bottoms:**
  $\frac{2}{3} + \frac{1}{5} \neq \frac{3}{8}$ ×
  Correct: find a common denominator first.
- **Flipping the wrong fraction:**
  $\frac{2}{3} \div \frac{4}{5} \neq \frac{3}{2} \times \frac{4}{5}$ ×
  Correct: Keep 1st, flip only the 2nd.

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Multiplying Fractions

Rule: Multiply tops, multiply bottoms.
No common denominator needed.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Steps:

1. Convert mixed numbers to improper fractions
2. **Cross-cancel** common factors first
3. Multiply across

e.g. $\frac{3}{8} \times \frac{4}{9}$: cancel 3 & 9, cancel 4 & 8
$\Rightarrow \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
By a whole number: write as $\frac{n}{1}$
e.g. $5 \times \frac{3}{7} = \frac{15}{7} = 2\frac{1}{7}$

Dividing Fractions (Keep–Change–Flip)

Rule: Multiply by the **reciprocal** of the 2nd fraction.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Steps:

1. Convert mixed numbers to improper fractions
2. **Keep** the 1st fraction
3. **Change** $\div$ to $\times$
4. **Flip** the 2nd fraction (reciprocal)
5. Cross-cancel, then multiply

e.g. $\frac{5}{6} \div \frac{10}{9} = \frac{5}{6} \times \frac{9}{10} = \frac{3}{4}$
Reciprocal: $\frac{a}{b} \times \frac{b}{a} = 1$

Order of Operations (BIDMAS)

Brackets → **I**ndices → **D**iv / **M**ult → **A**dd / **S**ub

Example 1: $\frac{2}{3} + \frac{1}{4} \times \frac{2}{5}$
Multiply first: $\frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$
Then add: $\frac{2}{3} + \frac{1}{10} = \frac{20}{30} + \frac{3}{30} = \frac{23}{30}$

Example 2: $\left(\frac{3}{5} + \frac{1}{3}\right) \div \frac{7}{15}$
Brackets: $\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$
Then divide: $\frac{14}{15} \times \frac{15}{7} = 2$

Fraction ⟷ Decimal Conversions

Frac → Dec: Divide numerator by denominator.
e.g. $\frac{13}{20} = 13 \div 20 = 0.65$

Dec → Frac: Write over power of 10, simplify.
e.g. $0.225 = \frac{225}{1000} = \frac{9}{40}$ (HCF = 25)

Common equivalences to memorise:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{8}$	$\frac{5}{8}$
0.5	$0.\overline{3}$	0.25	0.75	0.2	0.4	0.125	0.625

Word Problem Keywords & Key Formulas

Which operation do I use?

Keyword	Operation
"fraction of", "how much is"	Multiply
"how many pieces"	Divide
"total", "altogether"	Add
"remaining", "left over"	Subtract
"simplify", "lowest terms"	÷ HCF
"originally"	÷ remaining frac.

Essential formulas:

- **Fraction of amount:** $\frac{a}{b} \times$ total
- **Remaining fraction:** $1 -$ (fractions spent)
- **Working backwards:** Original = amount left ÷ frac. left
- **Reciprocal property:** $\frac{a}{b} \times \frac{b}{a} = 1$
- **Common denominator:** use LCM

Quick Exam Reminders

Always: Convert mixed → improper **before** calculating. Cross-cancel before multiplying. Simplify your final answer. Show every step for method marks!

Sense-check: Adding two fractions both $> \frac{1}{2}$? Answer must be > 1. Dividing by a proper fraction makes the answer **bigger**. Multiplying by a proper fraction makes it **smaller**.

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