Introduction to Algebra

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Variables & Constants

Letters represent unknown or changing values.

- **Variable:** A letter for an unknown (x, n, y).
- **Constant:** A fixed number (e.g. the 7 in $3x + 7$).
- **Coefficient:** The number before a variable (e.g. the 5 in $5x$).
- **Term:** A single piece separated by $+$ or $-$ signs.

Algebraic Conventions

Rules everyone agrees to follow:

- $3 \times x \to 3x$ (drop the $\times$ sign)
- $x \times x \to x^2$ (use index notation)
- $1x \to x$ and $-1x \to -x$
- Number before letter: $2a$ not $a2$
- Letters in alphabetical order: abc not cab
- $x \div 4 \to \frac{x}{4}$

Words $\to$ Algebra: Key Vocabulary

Operation	Keywords
$+$	sum, total, more than, increase
$-$	difference, less than, decrease
$\times$	product, times, double, triple, of
$\div$	quotient, share, split, half, per
x^n	squared, cubed, to the power of

Watch the order with subtraction!

- "7 less than n" $= n - 7$, **NOT** $7 - n$
- "n subtracted from 20" $= 20 - n$
- "Quadruple n, subtract 3" $= 4n - 3$
- "A third of n, add 8" $= \frac{n}{3} + 8$

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Substitution

Replace the letter with its value **in brackets**, then use BIDMAS.

- If $a = 5$, $b = -3$: $a^2 - 2b = (5)^2 - 2(-3) = 25 + 6 = 31$
- **Negative trap:** $(-4)^2 = 16$ but $-4^2 = -(4^2) = -16$
- **Always bracket negatives** to avoid squaring errors!

Simplifying: Like Terms

Only combine terms with the **same variable and same power**.

- $5x + 3x = 8x$ ✓
- $5x + 3x^2 \neq 8x^3$ ×

- $pq = qp$ (order doesn't matter)
- Remember: x means $1x$

Multiplying & Dividing Terms

Multiply: coefficients × coefficients, then add powers.
$4a \times 5b = 20ab$
$3x \times 7x = 21x^2$

Divide: coefficients ÷ coefficients, then subtract powers.
$18x^2y \div 6xy = 3x$

Forming Expressions from Problems

Steps:

1. Identify the unknown → assign a letter
2. Translate each relationship into algebra
3. Simplify where possible
4. Check with a test value

Common setups:

- Width $= w$, Length $= 2w - 3$
- Perimeter $= 2(\text{length} + \text{width})$
- Area $=$ length $\times$ width
- Patterns: find common difference d, then $dn + c$

Expression vs Equation vs Identity vs Formula

Type	Key Feature	Example
Expression	No equals sign	$8k - 3m + 2$
Equation	True for *specific* values	$7(x + 2) = 35$
Identity	True for *all* values ($\equiv$)	$5(a - 1) \equiv 5a - 5$
Formula	Relates variables; has a subject	$V = \frac{1}{3}\pi r^2 h$

Exam tip: No equals sign ⇒ it **cannot** be an equation, identity, or formula!

Quick Exam Shortcuts

Keyword spotting: "Write an expression for..." → form algebra. "Evaluate / find the value" → substitute. "Simplify" → collect like terms. "Find the nth term" → general expression in n.

Checking trick: Substitute a simple value (e.g. $x = 2$) into both your answer and the original to verify they match.

Consecutive even numbers: $2n$, $2n + 2$, $2n + 4$. **Consecutive odd:** $2n + 1$, $2n + 3$, $2n + 5$.

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