Linear Equations

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Linear Equations (The Basics)

$ax + b = c$ where the highest power of x is **1**.

- **Variable:** The unknown letter (x, y, etc.).
- **Coefficient:** The number before the variable (e.g. 3 in $3x$).
- **Constant:** A number on its own (e.g. 5 in $3x + 5$).
- **Solution:** The value that makes the equation true.

The Balance Principle

An equation is a balanced set of scales. Whatever you do to one side, do **exactly the same** to the other.

- $a = b \Rightarrow a \pm c = b \pm c$
- $a = b \Rightarrow a \times c = b \times c$
- $a = b \Rightarrow a \div c = b \div c \quad (c \neq 0)$
- **Goal:** Isolate the variable on one side.

The Four Inverse Operations

Every operation has an opposite. Use the inverse to "undo" and isolate x:

1. $x + a = b \Rightarrow x = b - a$

2. $x - a = b \Rightarrow x = b + a$

3. $ax = b \Rightarrow x = \dfrac{b}{a}$

4. $\dfrac{x}{a} = b \Rightarrow x = ab$

Two-step equations ($ax + b = c$): Undo the constant **first**, then undo the coefficient. Think: *BIDMAS backwards.*

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Brackets vs. Fractions: The Strategy

Brackets — Expand First

1. **Expand** using $a(b + c) = ab + ac$.
2. Simplify and collect like terms.
3. Solve as normal.

Watch: $-3(x - 4) = -3x + 12$
(negative × negative = positive!)

Fractions — Clear First

1. Find the **LCM** of all denominators.
2. Multiply **every term** on **both sides** by the LCM.
3. Fractions vanish — solve normally.

Watch: Multiply **all** terms, not just some!

Unknowns on Both Sides & Word Problems

Unknowns on both sides: Collect all x-terms on one side, constants on the other, then solve.

- Move x-terms to the side with the **larger coefficient** (avoids negatives).
- **Memory aid:** "Letters to one side, numbers to the other."

Word problems: **Read** twice → **Define** x → **Translate** to equation → **Solve** → **Answer** in words → **Check**.

"more than" $\rightarrow +$	"less than" $\rightarrow -$	"times / product" $\rightarrow \times$	"is / equals" $\rightarrow =$
"twice" $\rightarrow \times 2$	"half of" $\rightarrow \div 2$	"divided / shared" $\rightarrow \div$	"consecutive" $\rightarrow x, x+1, x+2$

Common Traps & Pitfalls

1. **The Inverse Trap:** $x + 5 = 14 \Rightarrow x = 14 - 5$, **not** $14 + 5$. Always use the *opposite* operation.
2. **The Bracket Trap:** $-3(x - 4) = -3x + 12$, **not** $-3x - 4$. Multiply **both** terms inside.
3. **The Fraction Trap:** Multiply **every** term by the LCM, on **both** sides. Missing one term breaks the balance.
4. **The Sign Trap:** When dividing by a negative, the sign of the answer flips: $-3x = -15 \Rightarrow x = +5$.
5. **The "No Check" Trap:** Always substitute your answer back. It catches careless errors and earns marks.

Quick Exam Shortcuts

Solving order: Constants first, coefficient second (BIDMAS backwards).
Both sides: Move x-terms to the side with the bigger coefficient to keep the coefficient positive.
Word problems: Underline numbers and key phrases. Define x *before* writing the equation.

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