Linear Simultaneous Equations

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What is a Simultaneous Equation?

A **pair of simultaneous equations** is two equations with the same unknowns that must **both be true at the same time**.

- Each equation in x and y represents a **straight line**.
- The **solution** is the (x, y) pair satisfying *both*.
- Geometrically: the **intersection point** of the two lines.
- A pair of linear equations has a unique, no, or infinite solutions.

Three Possible Outcomes

Lines...	Algebra gives...	Solutions
Intersect	One value each	**Unique**
Parallel	$0 = k$ $(k \neq 0)$	**None**
Identical	$0 = 0$	**∞ many**

Key check: always verify your (x, y) in *both* original equations after solving.

The Three Methods at a Glance

Method	When to use	Core idea	Watch out for
Substitution	One variable has coefficient ± 1	Rearrange $\rightarrow$ substitute $\rightarrow$ solve $\rightarrow$ back-substitute	Finding only one variable
Elimination	No coefficient is ± 1; coefficients can be matched easily	Multiply to match, then add (opp. signs) or subtract (same signs)	Sign errors when subtracting
Graphical	Question specifically asks for a graph	Plot both lines; read the intersection	Imprecise for non-integer answers

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Substitution Method: Steps

Step 1: Rearrange one equation to make x or y the subject.

Step 2: Substitute the expression into the *other* equation.

Step 3: Solve — it now has only **one** unknown.

Step 4: Back-substitute to find the second variable.

Step 5: Check in **both** original equations.

Example: $y = 3x + 1$, $2x + y = 11$
Sub: $2x + (3x + 1) = 11 \Rightarrow 5x = 10 \Rightarrow x = 2$
Back-sub: $y = 3(2) + 1 = 7$ **Solution:** $(2, 7)$

Elimination Method: Steps

Step 1: Label the equations (1) and (2).

Step 2: Multiply to make one variable's coefficients **equal in magnitude**.

Step 3: **Add** if signs are opposite; **Subtract** if signs are the same.

Step 4: Solve the resulting single-variable equation.

Step 5: Back-substitute to find the other variable.

Step 6: Check in **both** original equations.

Example: $3x + 4y = 16$, $5x - 2y = 18$
$\times 2$: $10x - 4y = 36$. Add: $13x = 52 \Rightarrow x = 4$

Back-sub: $y = 1$ **Solution:** $(4, 1)$

Word Problems: Strategy & Common Types

5-Step Strategy

Define the unknowns (with units)
↓
Form two equations
↓
Solve by any method
↓
Interpret in context
↓
Check in the original problem

Type	How to form the equations	Example
Cost/Price	$(\text{quantity}_1)(\text{price}_1) + (\text{quantity}_2)(\text{price}_2) = \text{total}$	$2a + c = 19$
Age	Current sum = total; future relationship = condition	$m + c = 50$; $m + 5 = 2(c + 5)$
Geometry	Use perimeter, angle sum, or area formula	$2l + 2w = 40$; $l = 3w$
Speed	$d = s \times t$ both ways; speeds add/subtract with current	$v + r = 30$; $v - r = 20$

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Common Traps & Pitfalls

1. **The Half-Answer Trap:** Finding x and stopping. *Always* find and state *both* unknowns, and verify both in *both* original equations.
2. **The Sign Trap (Subtraction):** When computing (1)−(2), *every* term in (2) changes sign. $(4x + y) - (4x - 3y) = 4y$, **not** $-2y$.
3. **The Fraction Trap:** When clearing fractions, multiply *every* term including the RHS. $\frac{x}{2} + \frac{y}{3} = 6 \times 6 \Rightarrow 3x + 2y = 36$, **not** $3x + 2y = 6$.

4. **The No-Rearrangement Trap:** If equations are not in standard form ($ax + by = c$), expand and rearrange *before* applying any method.
5. **The Same-Solution Trap:** Verify the answer makes sense in *context* for word problems (e.g. negative ages or prices are impossible).

Quick Exam Shortcuts & Special Cases

Fractions present? Multiply through by the LCD first: $\frac{x}{2} + \frac{y}{3} = 6 \xrightarrow{\times 6} 3x + 2y = 36$. $\frac{x}{3} + \frac{y}{9} = 3 \xrightarrow{\times 9} 3x + y = 27$. Then eliminate normally.

Equations need rearranging? Expand all brackets, collect x and y terms on the left, constants on the right, *then* solve.

Special cases (spotted during elimination): $0 = 4$ (or any $k \neq 0$) $\Rightarrow$ **no solution** (parallel lines). $0 = 0 \Rightarrow$ **infinitely many solutions** (same line, write the general form).

Quick self-check: After solving, substitute (x, y) into the equation you did *not* use last — if it balances, you're correct.

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