Percentages

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What is a Percentage?

$$p\% = \frac{p}{100}$$

- **Fraction → %:** divide, then × 100
- **% → Fraction:** write over 100, simplify
- **Decimal → %:** × 100 (shift 2 right)
- **% → Decimal:** ÷ 100 (shift 2 left)

Mental shortcuts:

- 10%: ÷10 5%: halve the 10%
- 1%: ÷100 25%: ÷4

Common Conversions to Memorise

Fraction	Decimal	%	Fraction	Decimal	%
$\frac{1}{2}$	0.5	50%	$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%	$\frac{1}{10}$	0.1	10%
$\frac{3}{4}$	0.75	75%	$\frac{1}{8}$	0.125	12.5%
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}\%$	$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}\%$

All Key Formulas

Skill	Formula	Key Rule
% of a quantity	$\frac{p}{100} \times Q$	Decimal × quantity
Express as %	$\frac{\text{Part}}{\text{Whole}} \times 100$	Same units. Part on top, whole on bottom
Increase / Decrease	Original × Multiplier	$+p\%$: $\times(1+\frac{p}{100})$ $-p\%$: $\times(1-\frac{p}{100})$
% Change	$\frac{\text{New} - \text{Original}}{\text{Original}} \times 100$	**Divide by ORIGINAL** (not new)
Reverse %	$\frac{\text{Given Value}}{\text{Multiplier}}$	Find what % the given value represents, then ÷
% Profit / Loss	$\frac{\text{Profit or Loss}}{\text{Cost Price}} \times 100$	**Divide by COST PRICE** (not selling price)

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Multiplier Quick Reference

Situation	Mult.	Reverse by
Increase by 5%	×1.05	÷1.05
Increase by 10%	×1.10	÷1.10
Increase by 20%	×1.20	÷1.20
Increase by 25%	×1.25	÷1.25
Decrease by 10%	×0.90	÷0.90
Decrease by 15%	×0.85	÷0.85
Decrease by 20%	×0.80	÷0.80
Decrease by 25%	×0.75	÷0.75

Successive % Changes

Apply multipliers **one at a time** to the updated value.

Successive changes ≠ their sum!

$$Q \times m_1 \times m_2 \neq Q \times (m_1 + m_2)$$

Profit & Loss

$$\%\ \text{Profit} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100 \quad \%\ \text{Loss} = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$$

Divide by **Cost Price (CP)**, never the selling price.

★ SOS: Common Mistakes to Avoid ★

Mistake 1: Wrong denominator in % Change

× Dividing by the *new* value

✓ Always divide by the **original** value

Correct: $\frac{57.60-48}{48}\times 100 = 20\%$ NOT $\frac{57.60-48}{57.60}$

Mistake 2: Reverse % — subtracting the %

× $598 - 30\% = 598 - 179.40 = \$418.60$

✓ $\frac{598}{1.30} = \$\mathbf{460}$

The % was applied to the *original*. Always divide by the multiplier.

Mistake 3: Wrong order — Part and Whole swapped

× $\frac{175}{56}\times 100 = 312.5\%$

✓ $\frac{56}{175}\times 100 = 32\%$

Part goes on top; *whole* goes on bottom.

Mistake 4: Adding successive %s together

× -25% then $-12\% = -37\%$ reduction

✓ $0.75 \times 0.88 = 0.66 =$ a 34% reduction

Multipliers must be *multiplied* together, not added.

Mistake 5: Profit/Loss — wrong denominator

× Dividing by the *selling* price

✓ Always divide by the **cost price**

$$\frac{34}{85}\times 100 = 40\%\text{ profit} \quad \text{NOT } \frac{34}{119}$$

Mistake 6: Units not matched

× $\frac{750}{2.5}\times 100 = 30{,}000\%$

✓ Convert first: $2.5\,\text{l} = 2500\,\text{ml}$, then $\frac{750}{2500}\times 100 = 30\%$

Keyword Spotting in Exam Questions

Exam phrase	What to do
"Find x% of ..."	Decimal × quantity
"What percentage is ... of ..."	Part ÷ whole ×100 (convert units first)
"Increase / decrease by x%"	Multiply by multiplier $(1 \pm \frac{p}{100})$
"Percentage change / increase / decrease"	$\frac{\text{New}-\text{Original}}{\text{Original}}\times 100$; original on the bottom
"Find the original price / value"	Reverse %: given ÷ multiplier
"Percentage profit / loss"	$\frac{\text{Profit or Loss}}{\text{Cost Price}}\times 100$
"Show that ..." (algebra)	Let original $= P$; apply multipliers, expand, simplify

Checking Your Answers

- **% of a quantity:** check that answer ÷ original = decimal
- **% Change:** re-apply the multiplier; the result should be the new value
- **Reverse %:** multiply your original by the multiplier — you should get back the given value
- **Profit/Loss:** a profit means SP > CP; a loss means SP < CP
- **Sanity check:** a 20% increase must give a number *larger* than the original; a decrease must give one *smaller*

Reverse % Step-by-Step

1. Decide whether the given value is **after an increase or decrease**
2. Calculate what % of the original the given value represents:
   After $+p$%: given $= (100+p)\%$ of original
   After $-p$%: given $= (100-p)\%$ of original
3. Convert that % to a decimal (the multiplier)
4. **Divide** the given value by the multiplier
5. Check: original × multiplier = given value ?✓

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