Polygons

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What is a Polygon?

A **polygon** is a **closed**, flat shape made of **straight** line segments that do not cross each other.

- **Sides:** The straight line segments (edges).
- **Vertices:** The points where two sides meet.
- **Rule:** Number of sides = number of vertices = number of angles.

Naming Polygons

3 Triangle	**8** Octagon
4 Quadrilateral	**9** Nonagon
5 Pentagon	**10** Decagon
6 Hexagon	**12** Dodecagon
7 Heptagon	n n-gon

Regular = all sides equal AND all angles equal.
Irregular = sides or angles (or both) differ.

The Five Key Polygon Formulas

For any polygon with n sides:

1. Sum of interior angles $= (n-2) \times 180°$
2. Each interior angle (regular) $= \dfrac{(n-2) \times 180°}{n}$
3. Each exterior angle (regular) $= \dfrac{360°}{n}$
4. Sum of exterior angles $= 360°$
5. Number of diagonals $= \dfrac{n(n-3)}{2}$

Key relationship: Interior angle + Exterior angle $= 180°$ (at every vertex)

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Angle Sums to Memorise

Polygon	n	Triangles	Angle Sum
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Nonagon	9	7	1260°
Decagon	10	8	1440°

Why? Split any polygon into $(n-2)$ triangles from one vertex.

Convex vs. Concave & Symmetry

Convex: All interior angles $< 180°$.
Concave: At least one reflex angle ($> 180°$); has a "dent".

Quick test: A rubber band around a convex polygon touches every vertex. If it skips one, the polygon is concave.

Symmetry of regular n-gons:

- n lines of symmetry
- Rotational symmetry of order n
- Angle of rotation $= \dfrac{360°}{n}$

Tessellation: Which Regular Polygons Tile a Plane?

For a regular polygon to tessellate on its own, its interior angle must divide exactly into 360°.

Shape	Angle	360° ÷ angle	Tiles?	Shape	Angle	360° ÷ angle	Tiles?
Eq. triangle	60°	6	Yes	Pentagon	108°	3.33...	No
Square	90°	4	Yes	Octagon	135°	2.67...	No
Hexagon	120°	3	Yes				

Only three: equilateral triangle, square, and regular hexagon.

Common Traps & Pitfalls

1. **Regular ≠ equal sides only:** A rhombus has 4 equal sides but is **not** regular (angles differ). Both conditions needed!
2. **Interior vs. exterior mix-up:** Read the question carefully. They are different values; don't give one when the other is asked.
3. **Forgetting to find n via exterior angles:** The fastest route to n is often: exterior angle $= 180° -$ interior angle, then $n = 360° \div$ exterior angle.
4. **Algebra errors with angle sums:** Always check your angles add up to $(n-2) \times 180°$ at the end.
5. **Confusing diagonals formula:** It's $\frac{n(n-3)}{2}$, **not** $\frac{n(n-1)}{2}$. Each vertex connects to $n-3$ others (not itself, not its 2 neighbours).

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Quick Exam Shortcuts

Finding number of sides from an interior angle: Exterior angle $= 180° -$ interior angle, then $n = 360 \div$ exterior angle. Faster than solving $\frac{(n-2)\times 180}{n} = \theta$.

Missing angle in an irregular polygon: Angle sum − sum of known angles = missing angle.

Polygons meeting at a point: Angles around a point sum to 360°. Set up: $\theta_1 + \theta_2 + \cdots = 360°$.

Interior–exterior ratio: If interior $= k\times$ exterior, then $(k+1) \times \text{ext} = 180°$, so ext $= \frac{180}{k+1}$, then $n = \frac{360}{\text{ext}}$.