Powers & Roots

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Powers (The Repeated Multipliers)

a^n means a multiplied by itself n times.

- **Base:** The number being multiplied (a).
- **Index:** How many times (n). Also called *exponent* or *power*.
- **Signs:** $(\text{negative})^{\text{even}} = \text{positive}$, $(\text{negative})^{\text{odd}} = \text{negative}$.

Roots (The Inverse Operation)

$\sqrt[n]{a}$ is the value that, raised to n, gives a.

- **Square root:** $\sqrt{a} \geq 0$ (positive only). Write $\pm\sqrt{a}$ for both.
- **Cube root:** $\sqrt[3]{a}$ can be negative.
- **Key:** $\sqrt{\text{neg}}$ is not real, but $\sqrt[3]{\text{neg}}$ is fine.

The Seven Laws of Indices

For any non-zero base a and integers m, n:

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1 \quad (a \neq 0)$
5. $a^{-n} = \dfrac{1}{a^n}$
6. $a^{1/n} = \sqrt[n]{a}$
7. $a^{m/n} = (\sqrt[n]{a})^m$
8. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

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Fractional Indices vs. Standard Form: The Strategy

Feature	Fractional Indices	Standard Form
Form	$a^{m/n}$	$a \times 10^n$ $(1 \leq a < 10)$
Method	**R-R-P**	Move decimal point
Key	Root first, power second	$n>0$ large, $n<0$ small

R-R-P Method

Reciprocal
↓
Root
↓
Power

Surds: Quick Rules

A surd is a root that **cannot** be simplified to a rational number (e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$).

- **Simplify:** Find the largest perfect square factor: $\sqrt{n} = \sqrt{k^2 \times m} = k\sqrt{m}$.
- **Multiply/Divide:** $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{a/b} = \sqrt{a} \,/\, \sqrt{b}$.
- **Add/Subtract:** Only **like surds** combine: $a\sqrt{n} \pm b\sqrt{n} = (a \pm b)\sqrt{n}$.

Common Traps & Pitfalls

1. **The Bracket Trap:** $(-3)^4 = +81$ but $-3^4 = -81$. Brackets change everything.
2. **The Zero Trap:** $a^0 = 1$, **not** 0. Also: $6 \times 3^0 = 6 \times 1 = 6$, not $(6 \times 3)^0 = 1$.
3. **The Coefficient Trap:** $(2a^3)^4 = 2^4 \times a^{12} = 16a^{12}$, not $2a^{12}$. Raise **everything** inside.
4. **The Surd Trap:** $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. Only multiplication splits under the root.
5. **The Standard Form Trap:** Always check $1 \leq a < 10$ in your final answer.

Quick Exam Shortcuts

Solving index equations: Write both sides with the **same base**, then equate the indices (e.g. $5^{3x} = 625 \Rightarrow 5^{3x} = 5^4 \Rightarrow 3x = 4$).
Growth/Decay: $A = P(1 \pm r)^n$ (+ for growth/interest, − for depreciation; r as a decimal).
Numbers to know: Squares up to $20^2 = 400$. Cubes up to $10^3 = 1000$. Powers of 2 up to $2^{10} = 1024$.

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