Probability

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Core Definitions

- **Trial:** A single experiment (e.g. rolling a die).
- **Outcome:** One possible result of a trial.
- **Event:** A set of outcomes we care about.
- **Sample space (S):** All possible outcomes.
- **Probability:** Always $0 \leq P(A) \leq 1$.

Theoretical vs. Experimental

Theoretical: $P(A) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$

Only works when all outcomes are equally likely.

Experimental: $P(A) \approx \dfrac{\text{times } A \text{ occurred}}{\text{total trials}}$

Gets closer to theoretical as trials increase.

Expected frequency $= P(A) \times n$

The Key Probability Rules

1. $P(A') = 1 - P(A)$ *(Complement)*
2. $P(A \text{ or } B) = P(A) + P(B)$ *(Mut. Excl.)*
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ *(General)*
4. $P(A \text{ and } B) = P(A) \times P(B)$ *(Independent)*
5. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ *(Conditional)*
6. $P(\text{at least one}) = 1 - P(\text{none})$

Keywords & Diagram Selection: The Strategy

Keyword ⟶ Action

Keyword	What to do
"OR"	**ADD** probabilities
"AND"	**MULTIPLY** probabilities
"NOT"	Complement $(1 - P)$
"AT LEAST ONE"	$1 - P(\text{none})$
"GIVEN THAT"	Conditional: new denom.

Diagram ⟶ When to use

Diagram	Best for
Sample space	Two events at once (e.g. 2 dice)
Tree diagram	Sequential / replacement
Venn diagram	Overlapping groups / sets
Two-way table	Conditional / given that

Tree Diagrams & Venn Diagrams: Quick Rules

Tree Diagram Rules:

- **Multiply along** branches for one path.
- **Add between** paths for combined outcomes.
- Branches from any point must **sum to 1**.

	With repl.	Without repl.
Probs	Stay the **same**	**Change**
Total	Unchanged	Drops by 1
Events	Independent	Dependent

Venn Diagram Method:

Step 1: Fill in the **overlap** first

↓

Step 2: Subtract overlap from each circle

↓

Step 3: Total − all regions = neither

Set notation:

$A \cup B$ = or $A \cap B$ = and A' = not A

Common Traps & Pitfalls

1. **The Replacement Trap:** Without replacement ⇒ probabilities **change** on the second draw!
2. **The OR/AND Trap:** "or" = ADD, "and" = MULTIPLY. The #1 source of errors.
3. **The Overlap Trap:** $P(A \text{ or } B) = P(A) + P(B)$ only if mutually exclusive. Otherwise, subtract $P(A \cap B)$.
4. **The Conditional Trap:** "Given B" ⇒ denominator is $n(B)$, **not** the total sample space.
5. **The Range Trap:** Answer < 0 or > 1? You have made an error. Always check!

Quick Exam Shortcuts

"At least one" shortcut: Always use $1 - P(\text{none})$. Never try to list every case individually.
Symmetry trick: For "exactly one of each," both orderings often give the same product — just double it.
Quick check: All probs in a complete sample space sum to **exactly 1**. Use this to find missing values.

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