Pythagoras' Theorem

Prepared by Nikolas Agisilaou

Right-Angled Triangles

A triangle with exactly one 90 angle.

- **Hypotenuse:** The longest side, always *opposite* the right angle.
- **Legs:** The two shorter sides that form the right angle.
- **Spotting it:** Look for the small square symbol in the corner of the triangle.

The Theorem

In any right-angled triangle with hypotenuse c and legs a, b:

$$\boxed{a^2 + b^2 = c^2}$$

- **Find hypotenuse:** $c = \sqrt{a^2 + b^2}$ (ADD)
- **Find a leg:** $a = \sqrt{c^2 - b^2}$ (SUBTRACT)
- **Only** works for right-angled triangles!

Pythagorean Triples (Memorise These!)

Sets of whole numbers (a, b, c) where $a^2 + b^2 = c^2$. Any **multiple** of a triple is also a triple.

(3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25)

Tip: Spot scaled triples instantly! E.g. sides 14 and 48 $\rightarrow 2 \times (7, 24, 25) \rightarrow$ hypotenuse is 50.

Problem-Solving Strategy

You see...	You do...	You see...	You do...
"Hypotenuse / diagonal"	$c = \sqrt{a^2 + b^2}$	"Is it right-angled?"	Check $a^2 + b^2 \stackrel{?}{=} c^2$
"Height / shorter side"	$a = \sqrt{c^2 - b^2}$	"3D / cuboid / pyramid"	Pythagoras **twice**
"Area of a triangle"	Find h first, then $\frac{1}{2}bh$	**Converse:** $a^2+b^2 > c^2$	Acute triangle
		Converse: $a^2+b^2 < c^2$	Obtuse triangle

Key Applications

2D Shapes

- **Isosceles height:** Drop perpendicular; base splits in half. $h = \sqrt{a^2 - (b/2)^2}$
- **Equilateral height:** $h = \frac{s\sqrt{3}}{2}$
- **Rectangle diagonal:** $d = \sqrt{l^2 + w^2}$

3D Shapes

- **Cuboid space diagonal:** $d = \sqrt{l^2 + w^2 + h^2}$
- **Pyramid slant height:** Find horizontal distance from centre to edge midpoint, then Pythagoras with vertical height.
- **Key trick:** Keep intermediate values as x^2 to avoid rounding errors.

Common Traps

1. **Add/Subtract:** Finding a leg? **Subtract**: $a^2 = c^2 - b^2$.
2. **Hypotenuse:** Identify it *first* — it's opposite the right angle.
3. **Rounding:** Only round at the **final** step. Keep x^2 values exact in 3D.
4. **"Looks right-angled":** Never assume. Use the converse test.
5. **Surds:** Exact answer $= 7\sqrt{3}$, not 12.1.

Quick Exam Shortcuts

- **Triple spotting:** Check if known sides match a triple before calculating.
- **Equilateral:** $h = \frac{s\sqrt{3}}{2}$, Area $= \frac{s^2\sqrt{3}}{4}$.
- **3D in one step:** Cuboid diagonal $= \sqrt{l^2 + w^2 + h^2}$.
- **Verify:** Substitute all three sides back into $a^2 + b^2 = c^2$.

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