Quadratic Expressions

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Quadratic Expression (The Building Block)

$ax^2 + bx + c$ where $a \neq 0$.

- a: Leading coefficient (of x^2).
- b: Coefficient of linear term (x).
- c: Constant term (no x).
- **Expression $\neq$ Equation**. No equals sign!

Expanding vs. Factorising (Reverse Operations)

Expanding: Brackets $\rightarrow$ no brackets.
$(x+4)(x+9) = x^2 + 13x + 36$
Factorising: No brackets $\rightarrow$ brackets.
$x^2 + 13x + 36 = (x+4)(x+9)$

- Always **check** by expanding back.

The Expansion Toolkit

1. $(x+a)(x+b) = x^2 + (a+b)x + ab$
2. **FOIL:** First, Outer, Inner, Last
3. Coeff. of x = **sum**; constant = **product**
4. $(a+b)^2 = a^2 + 2ab + b^2$
5. $(a-b)^2 = a^2 - 2ab + b^2$
6. $A^2 - B^2 = (A+B)(A-B)$

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Factorising: The Two Key Methods

Feature	When $a = 1$	When $a \neq 1$
Form	$x^2 + bx + c$	$ax^2 + bx + c$
Find	$p + q = b$, $pq = c$	Add to b, multiply to ac
Method	Write $(x+p)(x+q)$	Split middle, group, factor

Sign Guide ($a = 1$)

$c > 0$, $b > 0$: both $+$
$c > 0$, $b < 0$: both $-$
$c < 0$: one $+$, one $-$

The Factorisation Flowchart

"**Factorise completely**" = keep factorising until **every** factor is prime.

- **Step 1 — Common factor:** Take out the HCF of all terms first. Always.
- **Step 2 — Two terms?** Check for difference of two squares: $A^2 - B^2 = (A+B)(A-B)$.
- **Step 3 — Three terms?** Perfect square trinomial? If not, use sum/product or splitting.
- **Step 4 — Four terms?** Group into pairs, factorise each, extract the common bracket.

Common Traps and Pitfalls

1. **The Squaring Trap:** $(x+6)^2 = x^2 + 12x + 36$, **not** $x^2 + 36$. Never forget the $2ab$ middle term.
2. **The Sign Trap:** $(-4)(-8) = +32$, not -32. Negative $\times$ negative = positive.

3. **The Sum-of-Squares Trap:** $x^2 + 25$ **cannot** be factorised. DOTS only works with **minus**.
4. **The HCF Trap:** Always extract the common factor **before** harder methods.
5. **The "Completely" Trap:** $5x^2 - 80 = 5(x^2 - 16) = 5(x + 4)(x - 4)$ — keep going!

Quick Exam Shortcuts

Speed check: Constant = product of bracket numbers; x-coefficient = their sum.
Three brackets: Expand any **two** first, then multiply the result by the third bracket.
Two variables: FOIL works identically — e.g. $(3x + 2y)(x - 5y) = 3x^2 - 13xy - 10y^2$.
Perfect squares to know: $1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225$.

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