Ratio, Rate & Proportion

Prepared by Nikolas Agisilaou

Ratio

Definition: Comparison of *same-unit* quantities — **no units** in the answer.

Simplify:

- Convert all quantities to the **same unit** first
- Divide every part by the **HCF**
- Decimals: $\times 10^n$ to clear Fractions: $\times$ LCM of denominators

$24 : 36 \xrightarrow{\div 12} 2 : 3 \quad 1.5 : 4.5 \xrightarrow{\times 10} 15 : 45 \xrightarrow{\div 15} 1 : 3$

$\frac{3}{4} : \frac{5}{8} \xrightarrow{\times 8} 6 : 5$

Dividing Q in ratio $a : b$ (or $a : b : c$):

1. Total parts $= a + b$ (add all parts for three-way)
2. One part $= Q \div (a + b)$
3. Each share = ratio number $\times$ one part

Missing value: Divide known amount by *its* ratio number to get one part; multiply by the other.

Rate & Speed–Distance–Time

Rate: Comparison of *different*-unit quantities — answer **carries units**.

Unit rate: Divide to find value per 1 unit. Compare by finding cost per ml, per g, etc.

$D = S \times T$ find **distance**

$S = D \div T$ find **speed**

$T = D \div S$ find **time**



Cover the unknown!

Unit check: Speed in km/h $\Rightarrow$ time in **hours**, dist. in **km**.

Convert: mins $\div$ 60 = hours.

Kinematic Graphs

	Distance–Time (d–t)	Velocity–Time (v–t)
Gradient equals	Speed (velocity)	Acceleration
Area under equals	—	**Distance travelled**
Horizontal line	Object **stopped**	**Constant speed** (not stopped!)
Straight slope	Constant speed	Constant acceleration
Curve upward	Speeding up	Increasing acceleration
Downward slope	Returning to start	Decelerating

v–t area shapes:

Triangle (accel. from/to 0): $\frac{1}{2}bh$

Rectangle (constant velocity): bh

Trapezium (between two speeds): $\frac{1}{2}(a+b)h$

$$\text{Acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{Speed} = \frac{\Delta d}{\Delta t}$$

The Panda Math Tutor

Direct Proportion

$y \propto x \Rightarrow \boxed{y = kx}$ graph: **straight line through origin**

$\frac{y}{x} = k$ is **constant** (more $x \rightarrow$ more y)

Other forms:

$y \propto x^2 \Rightarrow y = kx^2 \qquad y \propto x^3 \Rightarrow y = kx^3$

$y \propto \sqrt{x} \Rightarrow y = k\sqrt{x}$

Method: Write $y = kx^n \rightarrow$ sub. known pair to find $k \rightarrow$ use equation.

Inverse Proportion

$y \propto \frac{1}{x} \Rightarrow \boxed{y = \frac{k}{x}}$ graph: **hyperbola** (never touches axes)

$x \times y = k$ is **constant** (more $x \rightarrow$ less y)

Other forms:

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2} \qquad y \propto \frac{1}{\sqrt{x}} \Rightarrow y = \frac{k}{\sqrt{x}}$$

Method: Write $y = k/x^n \rightarrow$ find $k = x^n \times y \rightarrow$ use equation.

Percentage as Proportion

$p\% = \dfrac{p}{100}$ (percentage is a ratio out of 100)

% of a quantity: $\dfrac{p}{100} \times Q$

Express as %: $\dfrac{\text{part}}{\text{whole}} \times 100$

% change: $\dfrac{\text{change}}{\text{original}} \times 100$

Increase by r%: multiply by $(1 + r/100)$

Decrease by r%: multiply by $(1 - r/100)$

Reverse %: original $= \dfrac{\text{new value}}{\text{multiplier}}$

Map Scales & Scale Drawings

$\text{Scale} = \dfrac{\text{drawn length}}{\text{actual length}}$

Scale $1 : n$ means 1 unit drawn $\equiv n$ units real.

Actual = map $\times\, n$ Map = actual $\div\, n$

Applying a scale factor k:

Length	Area	Volume
$\times\, k$	$\times\, k^2$	$\times\, k^3$

e.g. scale $1 : 40$*: area factor* $= 40^2 = 1600$*, volume factor* $= 40^3 = 64\,000$

Common Traps & Pitfalls

1. **Units Trap:** Convert to same unit *before* writing a ratio. $50\,\text{cm} : 2\,\text{m} = 50 : 200 = 1 : 4$, **not** $50 : 2$.
2. **Direct vs Inverse:** Direct $\Rightarrow$ ratio $y/x = k$. Inverse $\Rightarrow$ product $xy = k$. Mixing these up gives a completely wrong answer.
3. **Graph Trap:** Horizontal on d–t = **stopped**. Horizontal on v–t = **constant speed**. Same shape; opposite meanings!
4. **Area Trap:** Area under v–t gives **distance (metres)**. Decompose into triangles, rectangles, trapezia — do **not** count squares.
5. **Reverse % Trap:** To recover the original, **divide** by the multiplier — never subtract the % from the new value.
6. **Scale Factor Trap:** Length factor k gives area k^2 and volume k^3. Never apply k to all three.

Quick Exam Keywords

"Share / split / in ratio $a : b$" $\rightarrow$
Ratio division: find one part first.

"Per / each / best value" $\rightarrow$
Unit rate: divide to per-1-unit.

"More... more" / "directly prop." $\rightarrow$
Direct: $y = kx^n$, find k from pair.

"More... less" / "inversely prop." $\rightarrow$
Inverse: $xy = k$, use $y = k/x^n$.

"Find original after % change" $\rightarrow$
Reverse %: new $\div\, (1 \pm r/100)$.

"Gradient" $\rightarrow$ d–t: speed; v–t: accel.
"Area under" $\rightarrow$ v–t only: distance.

"Map / scale / model" $\rightarrow$
Scale $1 : n$; area k^2; volume k^3.

Key Formulas at a Glance

Ratio	Proportion	Kinematics	Percentage
One part $= Q \div (a+b)$	Direct: $y = kx^n$, $\dfrac{y}{x} = k$	$S = \dfrac{D}{T}$, $D = ST$, $T = \dfrac{D}{S}$	% change $= \dfrac{\Delta}{\text{orig}} \times 100$
Simplify by HCF	Inverse: $y = \dfrac{k}{x^n}$, $xy = k$	Accel $= \dfrac{\Delta v}{\Delta t}$	Reverse: orig $= \dfrac{\text{new}}{1 \pm r}$
Scale: drawn $\div$ actual	Find k from a known pair	Area under v–t = distance	$p\% = p/100$

The Pure Math Tutor