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Rational and Irrational Numbers

Solutions & Answer Key

Full worked solutions, marking scheme & answer key

IGCSE Mathematics

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Companion to the *Student Worksheet*

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Note

About this booklet. These are the complete solutions to every question in the *Rational and Irrational Numbers — Student Worksheet*. Question numbers match exactly: the number system N1–N6, rational numbers and their decimals T1–T7, recurring decimals D1–D7, irrational numbers I1–I7, operations O1–O6, applications A1–A5, the Mixed Review MR1–MR10, and the examination questions Q1–Q8 of Part III.
Each solution names the definition or property being applied and works one step at a time. Part V collects the common mistakes to watch for, and Part VI is a quick answer key for fast marking.

4 Part IV — Complete Solutions

Note

Each solution states the reason as well as the answer, because in this topic almost every mark is a *reasoning* mark. Depth is scaled to difficulty — brief for basic questions, fuller for the harder ones.

4.1 Solutions — The Number System

Solution N1

(a) An integer is a whole number, positive, negative or zero. Note that $\sqrt{49} = 7$, which is a whole number:

$$\boxed{-6, \quad \sqrt{49}\ (=7), \quad 0}$$

(b) 15 is not a perfect square, and any non-zero multiple of π is irrational:

$$\boxed{\sqrt{15}, \quad \tfrac{\pi}{2}}$$

The remaining numbers, $\frac{3}{4}$ and 2.7, are rational but not integers.

Solution N2

A rational number is any number of the form $\dfrac{p}{q}$ with p and q integers and $q \neq 0$. Every integer can be written with a denominator of 1:

$$13 = \frac{13}{1}, \qquad -13 = \frac{-13}{1}.$$

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Both are ratios of two integers with a non-zero denominator, so $\boxed{\text{every integer is rational}}$. This is why $\mathbb{Z} \subset \mathbb{Q}$.

Solution N3

(a) $\boxed{\text{True}}$ — the natural numbers are part of the integers ($\mathbb{N} \subset \mathbb{Z}$).
(b) $\boxed{\text{False}}$ — $\frac{1}{2}$ is rational but is not a whole number.
(c) $\boxed{\text{True}}$ — the real numbers are made up of the rationals together with the irrationals.
(d) $\boxed{\text{False}}$ — every real number is either rational or irrational; there is no overlap and no gap.

Solution N4

Deal with the disguised numbers first: $\sqrt{64} = 8$, and $0.181818\ldots = 0.\dot{1}\dot{8} = \dfrac{2}{11}$.

Integers:	$-5, \quad \sqrt{64}\ (= 8)$
Rational but not integers:	$\dfrac{9}{2}, \quad 0.181818\ldots \left(= \dfrac{2}{11}\right)$
Irrational:	$\sqrt{31}, \quad 7\pi$

31 is not a perfect square, and a non-zero rational multiple of π is always irrational.

Solution N5

(a) For example $\boxed{\frac{3}{4}}$ — it is a ratio of two integers, but it is not a whole number.
(b) For example $\boxed{\sqrt{10}}$ — since $3^2 = 9$ and $4^2 = 16$, we have $9 < 10 < 16$, so $3 < \sqrt{10} < 4$, and 10 is not a perfect square so the root is irrational. ($\pi = 3.14159\ldots$ is also acceptable.)
(c) For example $\boxed{-2}$ — it is a whole number, but the natural numbers are the counting numbers $0, 1, 2, 3, \ldots$, which do not include negatives.

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