Rearranging Formulas

Prepared by Nikolas Agisilaou

What is the Subject?

The **subject** stands **alone** on one side of the equals sign (usually the left).

- In $A = bh$, the subject is A.
- **Rearranging** means rewriting so a *different* variable becomes the subject.
- Also called "**changing the subject**" or "making x the subject."

The Core Principle (Balance!)

A formula is a balanced scale — do the **same thing to both sides**.

- Use **inverse operations** to undo what acts on the variable.
- Work **outside-in**: peel away the outermost operation first.
- **Always check** by substituting your result back in.

Inverse Operations: The Toolkit

Every operation has an opposite that "undoes" it. Use the inverse to isolate the subject:

1. Addition $(+) \longleftrightarrow$ Subtraction $(-)$
2. Multiplication $(\times) \longleftrightarrow$ Division $(\div)$
3. x in numerator $\rightarrow$ multiply by denominator
4. x in denominator $\rightarrow$ multiply by it first
5. Squaring $\longleftrightarrow$ Square root $(\pm)$
6. Cubing $\longleftrightarrow$ Cube root
7. $x^2 = k \;\Rightarrow\; x = \pm\sqrt{k}$
8. Reciprocal: flip **entire** sides, not terms

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Subject Appears Once: Use Reverse BIDMAS

Situation	Action	Example
$+/-$ on x	Undo it	$y{=}x{-}11 \Rightarrow x{=}y{+}11$
$\times/\div$ on x	Undo it	$W{=}Fd \Rightarrow d{=}W/F$
x is squared	Root $(\pm)$	$A{=}\pi r^2 \Rightarrow r{=}\sqrt{A/\pi}$
x under root	Square it	$y{=}\sqrt{x{+}9} \Rightarrow x{=}y^2{-}9$
x in denom.	Mult. by it	$P{=}E/t \Rightarrow t{=}E/P$

Multi-Step Order

Undo $+$ / $-$

↓

Undo $\times$ / $\div$

↓

Undo powers / roots

(Reverse BIDMAS!)

Subject Appears More Than Once

When the variable you want appears on **both sides** (or twice on one side), you **cannot** simply use inverse operations. Instead, follow the **CFD** method:

1. **Expand** any brackets if needed.
2. **Collect** all terms containing the subject on one side; move everything else to the other.
3. **Factorise** — take the subject out as a common factor.

4. **Divide** both sides by the remaining bracket to isolate the variable.

Quick example: $ax + 5 = bx + 8 \;\Rightarrow\; ax - bx = 3 \;\Rightarrow\; x(a-b) = 3 \;\Rightarrow\; x = \dfrac{3}{a-b}$

Common Traps & Pitfalls

1. **The Partial Division Trap:** In $Q = 3m + 6n$, dividing by 3 gives $Q/3 = m + 2n$, **not** $m + 6n$. Divide **every** term.
2. **The Reciprocal Trap:** $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ does **not** mean $x = a + b$. Combine into one fraction *first*, then flip.
3. **The $\pm$ Trap:** If $x^2 = k$, then $x = \pm\sqrt{k}$. Context (lengths, radii, time) may rule out the negative root.
4. **The Cancel Trap:** Never divide both sides by the variable you want — you'll lose solutions. **Factorise** instead.
5. **The Sign Trap:** Moving $-3x$ to the other side makes it $+3x$. Always double-check sign changes.

Quick Exam Shortcuts

Key formulas you may need to rearrange:

$v = u + at$	(equations of motion)	$A = \pi r^2$	(area of circle)
$V = \frac{1}{3}\pi r^2 h$	(volume of cone)	$V = \frac{2}{3}\pi r^3$	(volume of hemisphere)
$S = \frac{n}{2}(a + l)$	(arithmetic series)	$T = 2\pi\sqrt{l/g}$	(pendulum period)

Decision checklist: Subject once? $\rightarrow$ Inverse ops. Subject twice? $\rightarrow$ Collect, factorise, divide. Fraction? $\rightarrow$ Clear it. Power/root? $\rightarrow$ Undo it.

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