Sequences

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Key Definitions

A **sequence** is an ordered list of numbers following a rule.

- **Term:** Each value in the list $(a_1, a_2, a_3, \ldots)$.
- **nth term (a_n):** Formula to find any term directly from its position n.
- **Common difference (d):** Constant added each time (arithmetic).
- **Common ratio (r):** Constant multiplied each time (geometric).

How to Identify the Type

Calculate differences and ratios to classify:

- **1st differences constant** $\rightarrow$ **Linear** (arithmetic).
- **2nd differences constant** $\rightarrow$ **Quadratic**.
- **Constant ratio** between consecutive terms $\rightarrow$ **Geometric**.
- **Each term = sum of previous two** $\rightarrow$ Fibonacci-type.

The Three Core nth Term Formulas

Linear	$a_n = a + (n-1)d \;=\; dn + c$	$d =$ common diff, $\quad c = a - d$
Quadratic	$a_n = an^2 + bn + c$	$a = \dfrac{\text{2nd difference}}{2}$, then find $bn + c$
Geometric	$a_n = a \cdot r^{\,n-1}$	$a =$ first term, $\quad r =$ common ratio

Finding the nth Term: Step-by-Step

Linear Sequence

1. Find $d = a_2 - a_1$
2. Write $a_n = dn + c$
3. Substitute $n = 1$ to find c
4. Check with another term

Tip: Coefficient of n = common difference.

Quadratic Sequence

1. Find 1st diffs, then 2nd diffs
2. n^2 coeff $= \frac{\text{2nd diff}}{2}$
3. Subtract an^2 from each term
4. Remainder is linear — find its nth term
5. Combine & check

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Special Sequences to Recognise

Square numbers:	$1, 4, 9, 16, 25, 36, 49, \ldots$	$a_n = n^2$
Cube numbers:	$1, 8, 27, 64, 125, 216, \ldots$	$a_n = n^3$
Triangular numbers:	$1, 3, 6, 10, 15, 21, 28, \ldots$	$T_n = \frac{n(n+1)}{2}$
Powers of 2:	$2, 4, 8, 16, 32, 64, 128, \ldots$	$a_n = 2^n$
Fibonacci:	$1, 1, 2, 3, 5, 8, 13, 21, \ldots$	$a_n = a_{n-1} + a_{n-2}$

Common Traps & Pitfalls

1. **The Exponent Trap:** Geometric uses r^{n-1}, **not** r^n. The 5th term uses r^4.
2. **The Halving Trap:** n^2 coefficient is **half** the 2nd difference. 2nd diff $= 6 \Rightarrow$ coeff $= 3$.
3. **The Membership Trap:** Solve $a_n = k$ for n. Only valid if n is a **positive integer**.
4. **The Sign Trap:** Negative $d =$ **decreasing** sequence. Watch signs when computing c.
5. **The Subtraction Trap:** Subtracting an^2 for quadratics — double-check negatives.

Quick Exam Shortcuts

Is k in the sequence? Set $a_n = k$, solve for n. Positive integer $\Rightarrow$ yes; otherwise $\Rightarrow$ no.

Missing terms: Arithmetic: $d = \frac{a_{\text{last}} - a_{\text{first}}}{\text{gaps}}$. Geometric: $r = \sqrt[\text{gaps}]{\frac{a_{\text{last}}}{a_{\text{first}}}}$.

Two sequences crossing: Set nth terms equal, solve, and test nearby integer values.

Know these: Squares to 15^2=225. Cubes to 6^3=216. Triangulars: 1, 3, 6, 10, 15, 21, 28, 36.

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