Sets

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Sets & Notation (The Language)

A **set** is a well-defined collection of **distinct** objects.

- **Element:** An object in a set. Written $a \in A$.
- **Not in:** $a \notin A$ means a is not in set A.
- **Size:** $n(A)$ = number of elements in A.
- **Key:** No repeats, order does not matter.

Types of Sets (The Building Blocks)

- **Universal set** $\mathcal{U}$ (or ξ): all elements under consideration.
- **Empty set** $\emptyset$ or $\{\}$: no elements. $n(\emptyset) = 0$.
- **Subset:** $A \subseteq B$ means every element of A is in B.
- **Proper subset:** $A \subset B$ means $A \subseteq B$ and $A \neq B$.
- **Key:** A set with n elements has 2^n subsets.

The Four Set Operations

For sets A and B within universal set $\mathcal{U}$:

1. $A \cup B$ (Union) — in A **or** B (or both)
2. $A \cap B$ (Intersection) — in **both** A **and** B
3. A' (Complement) — **not** in A
4. $A \setminus B$ (Difference) — in A but **not** B

Memory aid: $\cup$ looks like "U" for **U**nion (OR). $\cap$ looks like a hat that **narrows** things down (AND).

Complement facts: $A \cup A' = \mathcal{U}$, $A \cap A' = \emptyset$, $(A')' = A$, $n(A) + n(A') = n(\mathcal{U})$, $A \setminus B = A \cap B'$.

Venn Diagrams & Counting Formulas: The Strategy

Feature	Two Sets	Three Sets
Regions	4	8
Formula	$n(A \cup B) = n(A) + n(B)$ $- n(A \cap B)$	$n(A \cup B \cup C) = \sum n(\text{each})$ $- \sum n(\text{pairs}) + n(A \cap B \cap C)$
Check	All regions must sum to $n(\mathcal{U})$	

Filling Order

Centre (all overlaps)
↓
Two-set overlaps
↓
Single-set regions
↓
Outside $= n(\mathcal{U}) -$ rest

Shading & Combined Expressions: Quick Rules

Evaluating: Work **inside out** — brackets first, then complements ($'$), then unions ($\cup$) and intersections ($\cap$).

- **Union** ($\cup$): shade everything in **either** part. **Intersection** ($\cap$): shade only what is in **both**.
- **Complement** ($'$): **swap** shaded and unshaded regions.

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- **Key results:** $(A \cup B)'$ = outside both circles. $A' \cap B$ = part of B not in A. $(A \cap B)'$ = everything except overlap.

Common Traps & Pitfalls

1. **The Repeat Trap:** Sets have **no duplicates**. Letters of "TOFFEE" $= \{T, O, F, E\}$, not $\{T, O, F, F, E, E\}$.
2. **The Empty Set Trap:** $\emptyset \neq \{\emptyset\} \neq \{0\}$. First has 0 elements; second has 1; third has 1.
3. **The Symbol Trap:** $\cup$ means OR (combine everything), $\cap$ means AND (only overlap). Don't swap them!
4. **The Counting Trap:** $n(A \cup B) \neq n(A) + n(B)$. You **must subtract** $n(A \cap B)$ to avoid double-counting.
5. **The Venn Trap:** Always fill from the **centre outwards**, never from the outside in.

Quick Exam Shortcuts

"Neither" shortcut: $n(\text{neither}) = n(\mathcal{U}) - n(A \cup B)$. Find the union first, then subtract from total.
Probability from Venn: $P(\text{event}) = n(\text{favourable region}) \,/\, n(\mathcal{U})$. Just read the numbers off the diagram.
Algebraic Venn: If regions contain x, set up: *all regions* $= n(\mathcal{U})$, then solve for x.
Symbols to know: $\in$, $\notin$, $\cup$, $\cap$, $'$, $\subseteq$, $\subset$, $\emptyset$, $\mathcal{U}$, $n(A)$, $\setminus$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$.

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