Similarity & Congruence

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Congruent Shapes (Identical Copies)

Two shapes are **congruent** if they are exactly the same shape **and** size.

- **Sides:** All corresponding sides are equal.
- **Angles:** All corresponding angles are equal.
- **Notation:** $\triangle ABC \cong \triangle DEF$ (letter order matters!).
- **Transforms:** Reflections, rotations, translations only.

Similar Shapes (Scaled Copies)

Two shapes are **similar** if they have the same shape but not necessarily the same size.

- **Angles:** All corresponding angles are equal.
- **Sides:** All corresponding sides are in the same ratio.
- **Scale factor:** $k = \dfrac{\text{larger length}}{\text{smaller length}}$.
- **Key:** All congruent shapes are similar ($k = 1$), but not vice versa.

Proving Congruence and Similarity in Triangles

Congruence (same shape AND size):

SSS	Three pairs of equal sides
SAS	Two sides + included angle equal
ASA	Two angles + included side equal
RHS	Right angle + hypotenuse + one side

Similarity (same shape, any size):

AA	Two pairs of equal angles
SSS (ratio)	All sides in the same ratio
SAS (ratio)	Two sides in ratio + included $\angle$equal

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Scale Factors: Length, Area, Volume

Quantity	Scale Factor	Example ($k = 3$)
Length	k	$\times 3$
Area	k^2	$\times 9$
Volume	k^3	$\times 27$

Going Backwards

Area SF known $\rightarrow k = \sqrt{\text{area SF}}$

$\downarrow$

Vol SF known $\rightarrow k = \sqrt[3]{\text{vol SF}}$

Problem-Solving Strategy

Congruence proof: Identify the two triangles $\rightarrow$ List matching sides/angles with reasons (e.g. "opposite sides of a parallelogram", "vertically opposite angles") $\rightarrow$ State condition (SSS, SAS, ASA, RHS) $\rightarrow$ Write congruence statement in correct letter order.

Similarity problem: Look for equal angles (parallel lines $\Rightarrow$ corresponding/alternate angles, shared angles, vertically opposite angles) $\rightarrow$ State condition (usually AA) $\rightarrow$ Write similarity statement in correct letter order $\rightarrow$ Set up and solve ratios of corresponding sides.

Common Traps and Pitfalls

1. **The AAA Trap:** AAA proves *similarity*, **not** congruence. Same angles $\neq$ same size.
2. **The SSA Trap:** Two sides and a *non-included* angle does NOT prove congruence. This is the "ambiguous case" — two different triangles can share the same SSA measurements.
3. **The Letter Order Trap:** $\triangle ABC \sim \triangle DEF$ means $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$. Wrong order = wrong ratios.
4. **The Scale Factor Trap:** Do NOT use k for area or k^2 for volume. Always: length $\rightarrow k$, area $\rightarrow k^2$, volume $\rightarrow k^3$.
5. **The Ratio Direction Trap:** Be consistent — always divide larger by smaller (or keep the same direction throughout your working).

Quick Exam Shortcuts

Parallel lines in a triangle: If $DE \parallel BC$ in $\triangle ABC$, then $\triangle ADE \sim \triangle ABC$ (AA). Use $\dfrac{AD}{AB} = \dfrac{DE}{BC}$.

Altitude to hypotenuse: In a right-angled triangle, the perpendicular from the right angle to the hypotenuse creates two triangles similar to each other *and* to the original. Key result: $AD^2 = BD \times DC$.

Shadow problems: Parallel sun rays $\Rightarrow$ similar triangles (AA). Set up: $\dfrac{\text{height}_1}{\text{shadow}_1} = \dfrac{\text{height}_2}{\text{shadow}_2}$.

Mass and capacity: For similar solids of the same material, mass and capacity scale by k^3.

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