Quadratic Equations

Prepared by Nikolas Agisilaou

What is a Quadratic Equation?

An equation of the form $ax^2 + bx + c = 0$.

- **Constants:** a, b, and c are numbers, with $a \neq 0$.
- **Solving:** Finding the unknown x values (roots/solutions) that make the equation true.
- **Requirement:** You need an equals sign and zero on one side to find solutions.

The Zero Product Property

If the product of two quantities is zero, at least one must be zero.

- **Rule:** If $A \times B = 0$, then $A = 0$ or $B = 0$.
- **Application:** Write the quadratic as $(x + p)(x + q) = 0$, then set each factor to zero.
- **Check:** Check your solutions by substituting back into the original equation.

Core Factorisation Methods

1. Monic ($a = 1$): To solve $x^2 + bx + c = 0$, find numbers p and q where $p + q = b$ and $p \times q = c$. Write as $(x + p)(x + q) = 0$.
2. Difference of Two Squares: $A^2 - B^2 = 0$ gives $(A + B)(A - B) = 0$. Solutions are $A = -B$ or $A = B$.
3. Perfect Square Trinomials: $(x + a)^2 = 0$ gives one repeated solution $x = -a$. This yields a repeated root because the factors are identical.

Non-Monic ($a \neq 1$) vs. Sign Guide

Splitting the Middle Term ($ax^2 + bx + c = 0$):

1. Calculate $a \times c$.
2. Find two numbers that add to b and multiply to ac.
3. Rewrite the middle term bx using those numbers.
4. Group into two pairs, factorise each pair, and take out the common bracket.

Sign Guide for $x^2 + bx + c$

c	b	**Signs of factors**
+	+	Both positive
+	−	Both negative
−	+	Larger is +, smaller is −
−	−	Larger is −, smaller is +

Rearranging Before Solving

Many quadratic equations are not given in standard form. You must rearrange so that one side equals zero.

- **Method:** Move all terms to one side, leaving 0 on the other.
- **Expansion First:** Expand brackets first (e.g., $x(x + 11) = -24 \rightarrow x^2 + 11x = -24$).
- **Common Factors:** Always check whether all terms share a common factor before using the splitting method.

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Common Traps & Pitfalls

1. **Not rearranging to zero first:** $x^2+4x=12$, so $(x)(x+4)=12$, therefore $x=12$ or $x+4=12$ is wrong! The zero product property ONLY applies when the product equals zero.
2. **Dividing both sides by x:** $x^2=7x$ divided by x gives $x=7$ which loses the solution $x=0$.
3. **Taking the square root too early:** Given $(x-2)^2=36$, writing $x-2=6$ misses the negative square root.
4. **Writing 'or' vs 'and':** Use 'or' because x takes one value or the other, not both at the same time.

Applied Problems (Word Problems)

Steps: Define a variable, form the equation, solve, then check context.
Rejecting Solutions: Reject negative values when the problem asks for lengths, areas, ages, or counts. Reject non-integer values when the problem asks for whole numbers. Always explain why you are rejecting a solution.

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