Straight Lines

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The Gradient (Slope)

The gradient m measures how steep a line is, showing the rise or fall for every horizontal unit.
Formula: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$.

- **Positive ($m > 0$):** Slopes upwards.
- **Negative ($m < 0$):** Slopes downwards.
- **Zero ($m = 0$):** Horizontal line.
- **Undefined:** Vertical line (division by zero).

Equation of a Straight Line

The gradient-intercept form is $y = mx + c$.

- **Gradient (m):** The slope of the line.
- **y-intercept (c):** The value of y where the line crosses the y-axis.
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$.
- **Horizontal Lines:** $y = k$ ($m = 0$).
- **Vertical Lines:** $x = k$ (m is undefined).

Parallel and Perpendicular Lines

Parallel Lines

Two lines are parallel if and only if they have the same gradient.

- **Rule:** $m_1 = m_2$.
- **Properties:** Parallel lines never meet and have different y-intercepts.

Perpendicular Lines

Two lines are perpendicular (meet at 90°) if the product of their gradients is -1.

- **Rule:** $m_1 \times m_2 = -1$.
- **Negative Reciprocal:** $m_2 = -\frac{1}{m_1}$. Flip the fraction and change the sign.

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Midpoint & Distance Formulas

The Midpoint Formula

The midpoint is simply the average of the x-coordinates and the average of the y-coordinates.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The Distance Formula

This formula comes from Pythagoras' theorem applied to the horizontal and vertical distances.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Sketching & Intersections

Sketching using the Intercept Method: Set $x = 0$ to find the y-intercept, set $y = 0$ to find the x-intercept, plot both, and draw a straight line through them.
Intersection of Lines: The point of intersection is where the lines cross, satisfying both equations simultaneously. Set equations equal ($m_1x + c_1 = m_2x + c_2$), solve for x, and substitute back to find y.
No Intersection: If lines are parallel, there is no intersection. If identical, there are infinitely many points.

Common Traps & Pitfalls

1. **Subtraction Order Trap:** When finding gradients, $m = \frac{y_2-y_1}{x_2-x_1}$ NOT $\frac{y_2-y_1}{x_1-x_2}$. You must be consistent in your order of subtraction.
2. **Rearranging Trap:** Always rearrange equations into $y = mx + c$ before reading off m and c.
3. **Perpendicular Trap:** The gradient is the **negative reciprocal**, not just the reciprocal (e.g., if $m = 5$, it is $-\frac{1}{5}$, not $\frac{1}{5}$).
4. **Distance Trap:** Do not forget to take the square root at the end of the distance formula. Also, squaring a negative yields a positive (e.g., $(-6)^2 = 36$, not -36).

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