The Quadratic Formula

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Standard Form of a Quadratic Equation

Every quadratic must be written as $ax^2+bx+c=0$, $a \neq 0$.

- a = coeff. of x^2 (never zero), b = coeff. of x, c = constant.
- **Always rearrange to zero first**, then read off a, b, c.
- **Watch signs:** if equation has $-7x$, then $b=-7$, not $+7$.

$5x^2-3x+7=0$ $a=5,\ b=-3,\ c=$
$4x^2=7x-2 \Rightarrow 4x^2-7x+2=0$ $a=4,\ b=-7,\ c=$
$8x-3x^2=5 \Rightarrow 3x^2-8x+5=0$ $a=3,\ b=-8,\ c=$

The Quadratic Formula

For $ax^2+bx+c=0$ $(a \neq 0)$:

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

- Gives **two** solutions: use $+$ then $-$ separately.
- **Rhyme:** *"neg b, plus/minus root b^2 minus $4ac$, all over $2a$."*
- **Derivation:** divide by a, complete the square, isolate x.
- Given in *some* exams — memorise it regardless.

The Discriminant $\Delta = b^2-4ac$ — Compute This FIRST

Condition	No. of Real Roots	Nature	Parabola
$\Delta > 0$	2 distinct real roots	Unequal values	Crosses x-axis at 2 points
$\Delta = 0$	1 repeated real root	Equal ($x=-b/2a$)	Touches x-axis at vertex only
$\Delta < 0$	No real roots	Not real	Lies entirely above or below x-axis

Finding unknowns: set $\Delta=0$ (equal roots), $\Delta>0$ (two roots), or $\Delta<0$ (no real roots), then solve for the unknown coefficient.

Exact Surd Answers

Leave in surd form when Δ is **not** a perfect square.

1. Compute $\Delta=b^2-4ac$.
2. Simplify $\sqrt{\Delta}$ by pulling out perfect square factors: $\sqrt{44}=\sqrt{4\times11}=2\sqrt{11}$; $\sqrt{28}=2\sqrt{7}$.
3. Cancel the common factor with $2a$: $\dfrac{8\pm2\sqrt{11}}{2}=4\pm\sqrt{11}$.

Rule: $\sqrt{ab}=\sqrt{a}\cdot\sqrt{b}$ **but** $\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$.

Graphical Interpretation

Roots of $ax^2+bx+c=0$ = x-intercepts of $y=ax^2+bx+c$.

- **Vertex x-coord:** $x_v=-\dfrac{b}{2a}$; substitute back for y_v.
- Axis of symmetry is **halfway** between the two roots.
- $a>0$: U-shape, minimum point; $a<0$: ∩-shape, maximum.
- **Check roots:** sum $=-\dfrac{b}{a}$; product $=\dfrac{c}{a}$.

Choosing the Right Method

Method	Use when...	Quick test / hint
Factorisation	Two integers multiply to ac and add to b	$x^2-7x+12$: need $\times=12$, $+=-7 \to -3$ and -4 ✓
Completing the Square	Need vertex form; or $a=1$ with b even	Gives $(x+\frac{b}{2})^2=k$; elegant for derivations
Quadratic Formula	**Always works**; non-factorisable, decimal or surd answer needed	Compute Δ first — if $\Delta<0$, stop immediately.

Common Traps & Pitfalls

1. **Sign trap:** $b=-5$ means $-b=+5$ and $b^2=(-5)^2=25$, not -25.
2. **$\pm$ trap:** Write **both** roots — omitting one loses marks.
3. **Fraction trap:** The *entire* numerator divides by $2a$, not just $\sqrt{\Delta}$.
4. **Rearrangement trap:** Set equation to zero **before** reading a, b, c.
5. **Context trap:** Reject impossible answers (e.g. negative length) and **explain why**.

Quick Exam Shortcuts

Step order: standard form → a, b, c → compute Δ → substitute → simplify → check.

Decimals: Round **only** at the final step; carry full precision through.

Equal roots ($\Delta=0$): single root is $x=-b/2a$, which equals the vertex x-coord.

Surd shortlist: know $\sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}, \sqrt{64}, \sqrt{100}, \sqrt{121}, \sqrt{144}$.

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