Sine & Cosine Rules

Prepared by Nikolas Agisilaou

Labelling Convention

Angles: capital letters A, B, C.
Sides: lowercase of the *opposite* angle.

- Side a opposite A; side b opposite B; side c opposite C.
- **Key:** longest side is always opposite the largest angle.

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- **Find side:** side on top. **Find angle:** sine on top.
- **Use when:** AAS (two angles + any side) *or* SSA (two sides + non-included angle).

The Cosine Rule & Area Formula

1. $a^2 = b^2 + c^2 - 2bc\cos A$

2. $b^2 = a^2 + c^2 - 2ac\cos B$

3. $c^2 = a^2 + b^2 - 2ab\cos C$

4. $\cos A = \frac{b^2+c^2-a^2}{2bc}$

5. $\cos B = \frac{a^2+c^2-b^2}{2ac}$

6. $\cos C = \frac{a^2+b^2-c^2}{2ab}$

7. Area $= \frac{1}{2}ab\sin C$ (C is the **included** angle between sides a and b; SAS only)

Use 1–3 (SAS): find *side*. **Use 4–6** (SSS): find *angle*. **Use 7** (SAS): find *area*. **Negative cosine** $\Rightarrow$ obtuse.

The Panda Math Tutor

Decision Guide & Ambiguous Case: The Strategy

Given	Find	Rule
Two angles + any side (AAS)	Side	Sine Rule
Two sides + non-incl. angle (SSA)	Angle/side	Sine Rule*
Two sides + incl. angle (SAS)	Side or area	Cosine Rule†
All three sides (SSS)	Angle	Cosine Rule

*Check ambiguous case! †Area: use formula 7 above.

Ambiguous Case

SSA: $\sin\theta = \sin(180°-\theta)$ may give **two** triangles.

Test $B_2 = 180° - B_1$; reject if $A + B_2 \geq 180°$.

Obtuse given angle $\Rightarrow$ one solution only.

Common Traps & Pitfalls

1. **The Angle Trap:** $\frac{1}{2}ab\sin C$ requires C to be the angle **between** sides a and b. Verify before substituting.
2. **The Ambiguous Trap:** When finding an angle from SSA, always check whether $180° - \theta$ also produces a valid triangle.
3. **The Sign Trap:** Obtuse angle $\Rightarrow$ negative cosine $\Rightarrow$ the $-2bc\cos A$ term becomes *positive*, making a^2 larger. This is geometrically correct.
4. **The Rounding Trap:** Keep full precision; only round at the final step — IGCSE mark schemes penalise premature rounding.

Quick Exam Shortcuts

Multi-step: Split shapes; carry exact values forward without rounding. **Largest angle first (SSS):** Find it first (no ambiguous case risk), then Sine Rule. **Sanity:** Angles = 180°, area > 0, longest side opp. largest angle.

The Panda Math Tutor