Triangles

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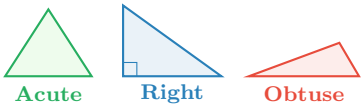
Types by Sides

- **Equilateral:** 3 equal sides, all angles 60.
- **Isosceles:** 2 equal sides; base angles are equal.
- **Scalene:** No equal sides, no equal angles.

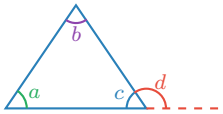


Types by Angles

- **Acute:** All angles < 90.
- **Right-angled:** One angle $= 90$.
- **Obtuse:** One angle > 90.



Angle Properties



$d = a + b$

- **Angle Sum:** $\angle A + \angle B + \angle C = 180$ (always!)
- **Missing angle:** $= 180 -$ (angle 1 + angle 2)
- **Exterior Angle Theorem:** An exterior angle equals the sum of the two **remote** interior angles.
- **Isosceles shortcut:** If two sides are equal, their opposite angles are equal.

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Area Formulas

1. Base & Height:

$$\text{Area} = \tfrac{1}{2} \times b \times h$$

h must be **perpendicular** to b.

2. Hero's (Heron's) Formula:

When all 3 sides a, b, c are known:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{a+b+c}{2}$ (semi-perimeter).

Pythagoras' Theorem

For **right-angled triangles** only:

$$c^2 = a^2 + b^2$$

- **Find hypotenuse:** $c = \sqrt{a^2 + b^2}$
- **Find shorter side:** $a = \sqrt{c^2 - b^2}$

Converse (classify a triangle):

- $c^2 = a^2 + b^2 \Rightarrow$ **Right**
- $c^2 < a^2 + b^2 \Rightarrow$ **Acute**
- $c^2 > a^2 + b^2 \Rightarrow$ **Obtuse**

Triangle Inequality + Traps & Exam Tips

Inequality: Sum of any two sides $>$ third side. **Shortcut:** two **shorter** sides must add to more than the **longest**. **Range:** If sides are x, p, q $(q > p)$, then $q - p < x < q + p$.

Common Traps:

1. Pythagoras ONLY for right-angled triangles.
2. Height in $\frac{1}{2}bh$ must be **perpendicular**.
3. Isosceles can be any orientation.
4. Exterior angle = sum of two *remote* angles.

Triples: $3, 4, 5$ — $5, 12, 13$ — $8, 15, 17$ — $7, 24, 25$

Keywords: "missing angle" → angle sum; "find the length" + right angle → Pythagoras; "find area" → pick formula.

Always: diagram, label, show working, units!