Trigonometry

Prepared by Nikolas Agisilaou

Labelling the Sides (relative to θ)

- **Hypotenuse (H):** Longest side; always opposite the right angle. Never changes.
- **Opposite (O):** Side directly facing angle θ.
- **Adjacent (A):** Side next to θ that is *not* the hypotenuse.

Warning: O and A **swap** when you change the reference angle. H never changes.

The Three Ratios: SOH CAH TOA

$\sin\theta = \dfrac{O}{H}$ **SOH**: Opposite ÷ Hypotenuse

$\cos\theta = \dfrac{A}{H}$ **CAH**: Adjacent ÷ Hypotenuse

$\tan\theta = \dfrac{O}{A}$ **TOA**: Opposite ÷ Adjacent

Finding a Missing Side

1. Label sides O, A, H relative to the given angle.
2. Identify the two sides involved (known + unknown).
3. Choose SOH, CAH, or TOA.
4. **Unknown on top:** multiply by trig ratio. **On bottom:** divide.

$\cos 36° = \frac{x}{8} \Rightarrow x = 8\cos 36° = 6.47$ cm $\quad \cos 42° = \frac{7.3}{h} \Rightarrow h = \frac{7.3}{\cos 42°} = 9.82$ cm

Finding a Missing Angle

Use **inverse trig** when two sides are known: $\theta = \sin^{-1}\left(\frac{O}{H}\right)$, $\theta = \cos^{-1}\left(\frac{A}{H}\right)$, $\theta = \tan^{-1}\left(\frac{O}{A}\right)$

1. Label sides relative to the *unknown* angle.
2. Identify the two known sides, choose the correct ratio.
3. Apply the inverse. Round to **1 d.p.** (IGCSE standard).

Tip: Third angle $= 90° - \theta$ (angles in triangle sum to 180°).

Exact Trigonometric Values — Must Memorise!

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$	Derived from
0°	0	1	0	Limiting case
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	Equilateral triangle halved: sides $1, \sqrt{3}, 2$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	Isosceles right triangle: legs $1, 1$; hyp $\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	Equilateral triangle halved: sides $1, \sqrt{3}, 2$
90°	1	0	undefined	Limiting case

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$ (always true for any angle)

Angles of Elevation & Depression

- **Elevation:** Angle *upward* from horizontal to an object above.
- **Depression:** Angle *downward* from horizontal to an object below.
- **Key:** Elevation A→B = Depression B→A (alternate angles).
- **Strategy:** Draw horizontal; form right triangle; usually use **TOA**.

Example: 58° elevation, 65 m away $\Rightarrow h = 65\tan 58° = 104.0$ m

Bearings and Trigonometry

- Measured **clockwise from North**; always **three digits** (e.g. 045°).
- Range: 000° to 360°.
- **cos** → North/South; **sin** → East/West.
- For bearings > 90°: extract the interior triangle angle first.

Example: 053°, 9.4 km: N= $9.4\cos 53° = 5.7$ km; E= $9.4\sin 53° = 7.5$ km

Trigonometry in 3D Problems

1. Identify the right-angled triangle you need.
2. Use **Pythagoras** to find an intermediate length (e.g. base diagonal).
3. Apply **SOH CAH TOA** to find the required side or angle.

Cuboid: base diag $d = \sqrt{l^2 + w^2}$, then $\tan\theta = \dfrac{h}{d}$

Pyramid: half base diag $= \dfrac{s\sqrt{2}}{2}$, then $\tan\theta = \dfrac{\text{height}}{\text{half diag}}$

Pythagoras vs. Trigonometry: Decision Guide

Use Pythagoras	Use SOH CAH TOA
Know 2 sides, find 3rd	Angle given or asked for
No angle given/needed	Know angle + 1 side
$a^2 + b^2 = c^2$	$\sin\theta, \cos\theta, \tan\theta$

Right angle? **No** → Sine/Cosine Rule.
Yes → Angle involved? **No** → Pythagoras. **Yes** → Trig.

Common Traps & Quick Exam Reference

Traps to Avoid:

1. **Wrong labels:** Always label O, A, H *first*, then pick ratio.
2. **Wrong mode:** Calculator must be in **Degree** mode, not Radians.
3. **Multiply vs. Divide:** Unknown on top → multiply; bottom → divide.
4. **Wrong angle:** Use the *given* angle, never the right angle.
5. **Early rounding:** Keep full values in working; round only at the final step.

Keyword Quick Reference:

- "Find side" + angle ⇒ SOH CAH TOA (multiply or divide)
- "Find angle" + two sides ⇒ $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$
- "Exact value" / "no calculator" ⇒ use the exact values table
- "Elevation/Depression" ⇒ draw horizontal; usually **tan**
- "Bearing" ⇒ clockwise from North; **sin**=E/W, **cos**=N/S
- "3D shape" ⇒ base diagonal (Pythagoras) first, then trig

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