Vectors & Transformations

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Vectors: The Basics

A **vector** has **magnitude** (size) *and* **direction**; a **scalar** has size only.

- **Notation:** $\overrightarrow{AB}$, bold $\mathbf{a}$ / underlined $\underline{a}$, or column $\binom{x}{y}$.
- **Column vector:** top = horizontal (+ right, − left); bottom = vertical (+ up, − down).
- **Between points:** $\overrightarrow{AB} = \binom{x_2-x_1}{y_2-y_1}$ (**end − start**).
- **Reverse:** $\overrightarrow{BA} = -\overrightarrow{AB}$ (same size, signs flip).

Magnitude & Scalar Multiples

Magnitude (length) by Pythagoras:

$$|\mathbf{a}| = \sqrt{x^2 + y^2} \quad (\geq 0)$$

- **Add / subtract:** componentwise, $\binom{a}{b} \pm \binom{c}{d} = \binom{a\pm c}{b\pm d}$.
- **Scalar:** $k\binom{x}{y} = \binom{kx}{ky}$; $k > 0$ same way, $k < 0$ reversed, length $\times|k|$.
- **Parallel:** $\mathbf{b} = k\,\mathbf{a}$ (one is a multiple of the other).

Position Vectors & Vector Geometry

Position vector of $P = (x, y)$: $\mathbf{p} = \overrightarrow{OP} = \binom{x}{y}$

Route rule (end − start): $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

Midpoint M of AB: $\mathbf{m} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

Collinear A, B, C**?** Show $\overrightarrow{AC} = k\,\overrightarrow{AB}$ (a scalar multiple) with a **shared point** $\Rightarrow$ they lie on one line. The value of k is the **length ratio** ($AC = k\,AB$).

The Four Transformations at a Glance

Transformation	What it does	Describe it fully with	Image is
Translation	slides (no turn/resize)	the column vector $\binom{x}{y}$	congruent
Reflection	flips over a mirror line	equation of the mirror line	congruent
Rotation	turns about a point	angle + direction + centre	congruent
Enlargement	resizes from a centre	scale factor k + centre	similar

Congruent = same size & shape (translation, reflection, rotation). **Similar** = same shape, different size (enlargement).

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Coordinate Rules (about the origin)

Reflections

Mirror line	Rule $(x, y) \to$
x-axis ($y = 0$)	$(x, -y)$
y-axis ($x = 0$)	$(-x, y)$
$y = x$	(y, x)
$y = -x$	$(-y, -x)$
$x = a$ (vertical)	$(2a - x, y)$
$y = b$ (horizontal)	$(x, 2b - y)$

Rotations & more

Transformation	Rule $(x, y) \to$
90° anticlockwise	$(-y, x)$
90° clockwise	$(y, -x)$
180° (either way)	$(-x, -y)$
270° anticlockwise	$(y, -x)$
Translate by $\binom{a}{b}$	$(x + a, y + b)$
Enlarge, centre C	$C + k(P - C)$

Common Traps & Pitfalls

1. **Direction Trap:** $\overrightarrow{AB} \neq \overrightarrow{BA}$; $\overrightarrow{BA} = -\overrightarrow{AB}$ (flip both signs).
2. **Negative-Square Trap:** in $|\mathbf{a}|$, $(-4)^2 = +16$, **not** -16. Length is never negative.
3. **Rotation Trap:** always give a **direction** — 90° clockwise and anticlockwise land in different places.
4. **Mirror-Line Trap:** $y = x$ swaps $(x, y) \to (y, x)$; $y = -x$ swaps *and* negates $(x, y) \to (-y, -x)$.
5. **Centre Trap:** enlargement distances are measured **from the centre**, not the origin (unless the centre *is* the origin).
6. **"Describe fully" Trap:** give **one** transformation with all its details — naming two usually scores zero.

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Quick Exam Shortcuts

Describe fully: Translation → vector; Reflection → mirror line; Rotation → angle + direction + centre; Enlargement → scale factor + centre.

Combining transformations: two reflections in **parallel** lines = a **translation**; two reflections in **intersecting** lines = a **rotation** about the crossing point; enlargement with $k = -1$ = a 180° **rotation** about the centre.

Working backwards: the **mirror line** is the perpendicular bisector of any point–image join; the **centre of enlargement** is where rays through matching vertices meet, with $k = \dfrac{\text{image length}}{\text{object length}}$ (negative k if the image is inverted).